Logical Foundations of Syllable Representations

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One key issue in all areas of theoretical linguistics is the best way of representing abstract structures. For example, a variety of syllabic structure representations have been employed in the phonological literature (e.g., [4, 5, 11]). A natural question is to what extent the differences between competing representations are significant or merely matters of notation. In this regard, this paper makes two contributions. First, it argues that model theory and logic [6] provide a foundation for studying questions of notational equivalence because the power of the logic needed to ‘translate’ between two representational structures is a measure of the meaningful difference between the representations. Second, it shows three distinct syllable representations are notationally equivalent in the strongest terms (that is, with the weakest logic), provided there is a bound on syllable size. This condition, while not universally adopted, has ample precedent in the phonological literature (e.g. [5]). It follows that practitioners can use any one of these representations studied because they are all easily intertranslatable. This result illustrates one type of insight model theory can provide for theoretical linguistics.

Intuitions are clear that font choice is irrelevant to the representation of a set of numbers: both \{1,2\} and \{1,2\} describe the same set. As practitioners, we abstract away from font choice when expressing such sets and confidently choose among them arbitrarily. However, it has been argued that different syllable representations are not interchangeable in the same way (see [8] for a review). If they are notationally equivalent, it stands to reason that there are no theoretically meaningful differences between them. Although they highlight different aspects of syllable structure, they may encode the same information. If they are not notationally equivalent, different predictions about cross-linguistic patterns could be made based on the types of constraints on syllable structure admitted by each type of representation.

Consider the following representations of the word \textit{planet}. The first simply demarcates syllable boundaries, as in [11, 17] and others. We call this the ‘dot’ representation due to the popularity of marking syllable boundaries with periods or dots. The remaining two representations refer to the syllable elements \textit{onset}, \textit{nucleus}, and \textit{coda}. We call (1b) the ‘flat’ representation. It treats syllable elements as secondary labels of each segment (as in [3, 10]). For example, the \textit{p} and \textit{l} in \textit{planet} are both labeled \textit{ons}. The representation in (1c) was introduced in [16] and subsequently argued for in [4] and others. It represents syllables hierarchically, with syllable nodes \sigma dominate syllable element nodes, which in turn dominate segment nodes. For this reason we call it the ‘tree’ representation.

\begin{enumerate}
  \item \text{a. ‘Dot’ representation:} a string of segments and syllable boundaries. \[.pla.net.\]
  \item \text{b. ‘Flat’ representation:} a string of segments labeled with syllable elements. \[
  \begin{array}{cccccccc}
  p & l & a & n & e & t \\
  \ons & \ons & \nuc & \ons & \nuc & \cod
  \end{array}
  \]
  \item \text{c. ‘Tree’ representation:} hierarchical structure. \[
  \begin{array}{cccccccc}
  \sigma & \sigma \\
  \ons & \nuc & \ons & \nuc & \cod \\
  \sigma & \sigma \\
  p & l & a & n & e & t
  \end{array}
  \]
\end{enumerate}

On one hand, the flat and tree models may seem superior because they refer directly to syllable elements. Yet it appears that the dot model has its own advantage over the flat model; marking syllable boundaries explicitly distinguishes two adjacent sounds that happen to be the same \textit{type} of element from two adjacent sounds that belong to the \textit{same} element of a single syllable, like the \textit{pl} onset in \textit{planet}. The tree model also makes this distinction. But is there a \textit{true} difference in the information encoded in each representation – or are these apparent differences illusory?

No previous literature addresses the question of whether these representations encode different information, but model theory provides a natural way to do just that [13, 14]. A representational structure or \textit{model} is a type of graph that encodes certain labels and relations. Each example in (1) shows how the word \textit{planet}
is represented in one of three models: the dot, flat, and tree models. A model \( M_1 \) is \( L \)-interpretable in terms of another, \( M_2 \), if one can write a graph transduction (in the sense of [7]) from \( M_1 \) to \( M_2 \) using logic \( L \). In a sense, a transduction is a way of translating information from one model to another. If \( M_1 \) is \( L \)-interpretable in terms of \( M_2 \) and vice versa, then we say the two are \( L \)-bi-interpretable.

For instance, one step in translating the dot model \( M_{dot} \) to the flat model \( M_{flat} \) would be to change a dot-C (consonant) sequence to C with the onset label, like the \( p \) in \( planet \). Comparing (1a) to (1b), it is easy to see that this is change is equivalent to the mapping in Figure 1. Similarly, translating from \( M_{flat} \) to \( M_{dot} \) requires that an onset C following a nucleus V (vowel) maps to a V-dot-C sequence, like the \( an \) in \( planet \). This mapping is exemplified in Figure 2. The paper presents full mappings for representing any word in the three models exemplified in (1). Note that we map each type of syllable representation onto the other two, and vice versa. We do not address the issue of mapping an unsyllabified string to a syllabified one; this is left to future work.

\[
\begin{array}{c}
\text{.p} 
\end{array} \quad \rightarrow \quad \begin{array}{c}
p \end{array}
\]

\[
\begin{array}{c}
a \quad n 
\end{array} \quad \rightarrow \quad \begin{array}{c}
a \cdot n 
\end{array}
\]

Figure 1: Partial mapping from \( M_{dot} \) to \( M_{flat} \). Figure 2: Partial mapping from \( M_{flat} \) to \( M_{dot} \).

Informally, \( L \)-bi-interpretability means the two models are notationally equivalent with respect to logic \( L \). It follows that the weaker the logic, the less meaningful the differences are between the models. Three logics in decreasing power are Monadic Second Order (MSO), First Order (FO), and Quantifier-Free (QF), which differ according to the types of allowable quantification [2, 6]. Many linguistic generalizations are known to be expressible in FO [9, 12, 13, 15] and QF characterizes a class of transductions important to phonology [1, 2].

This paper establishes as a mathematical fact that each pair of representation types exemplified in (1) are QF-bi-interpretable, provided there is a bound on the number of segments in a given nucleus, onset, or coda. Consequently, the same information is available in any of the models presented here; they cannot make different predictions regarding the types of syllable-based patterns available to the world’s languages. For example, any constraint on syllable structure expressed naturally in one of these formalisms is equally naturally expressible in the others. We expect future applications of model theory to shed light on the nature of other types of abstract linguistic representations in phonology, morphology, and syntax.