Real Options Switching Strategies in Dynamic Transport Service Operations

Qian-wen Guo\textsuperscript{a}, Joseph Y.J. Chow\textsuperscript{b,*}, Paul Schonfeld\textsuperscript{c}

\textsuperscript{a} Sun Yat-Sen University, Guangzhou, China
\textsuperscript{b} New York University, NY, USA
\textsuperscript{c} University of Maryland, College Park, MD, USA

INFORMS 2017, Houston, TX
Premise

- Need for policies to inform “when to switch” between two operational regimes:
  - Automation
    - Shared autonomous vehicle operations
    - Dynamic tolls/pricing (transit, freight, roads, parking, etc.)
    - Dynamic infrastructure use (traffic control, lanes, parking, etc.)
    - Dynamic fleet operations (dispatch, rebalancing, customer incentives, etc.)
  - Highly uncertain sequential decisions
    - New technology adoption
    - Rapidly growing/changing community
Our Contribution

Using (automated) last mile transit as primary application...

... we developed an optimal switching algorithm (available on GitHub) for data-driven decisions minimizing transit fleet operating costs

https://github.com/BUILTNYU/Optimal-Switching
Outline

- Background and literature review
- Problem definition
  - Problem illustration
  - Formal definition
- Proposed model
  - Dynamic switching between fixed and flexible transit
  - Model variation: modular vehicle sizes
- Model properties
- Computational evaluations
Background: Last Mile Transit Ops

Increasing need for optimization of automated, shared, on-demand transit to serve last mile

Dual-mode transit fleet operating strategies tend to be static

- Fixed route vs flexible service e.g. Kim and Schonfeld, 2013
- Vehicle size e.g. Fu and Ishkanov, 2004
- Headway control e.g. Thomas, 2007
- Idle vehicle relocation e.g. Yuan et al., 2011, Sayarshad and Chow, 2017
- Ridesharing options M to 1, M to Few, M to M. Daganzo, 1978; Chang and Schonfeld, 1991a,b; Quadrifoglio and Li, 2009
Research Gap

- Chang & Schonfeld (1991): optimize fixed route or flexible service for a last mile (M-to-1) region – threshold exists
- Kim & Schonfeld (2012): extended to multiple deterministic periods

No methodology to dynamically optimize switching between different operating states in last mile transit

Chang & Schonfeld (1991)

**Variable Representation**
- $ac_1$: average cost for fixed route service = $acu_1 + aco_1$
- $acu_1$: average user cost for fixed route service
- $aco_1$: average operator cost for fixed route service
- $ac_2$: average cost for flexible route service = $acu_2 + aco_2$
- $acu_2$: average user cost for flexible route service
- $aco_2$: average operator cost for flexible route service
Research Gap (2)

- Dixit (1989): Analytical expression for optimal switching timing for GBM stochastic process
- Sødal et al. (2009): applied to shipping with stochastic freight rates
- Tsekrekos (2010): using an infinite series (Kummer series) to derive a solution for stationary mean-reverting (Ornstein-Uhlenbeck) process

$$dQ = \mu(m - Q)dt + \sigma Qdw$$

Not yet applied to urban transport operations
Why not deterministic threshold?

- Hysteresis effect: presence of inertia


- Using deterministic threshold is a myopic policy
- Buffer can be designed to account for characteristics of stochastic process and cost of switching
Use cases

- Switching between fixed route and on-demand

- Switching between one-module and two-module vehicles (vehicle “size”)
Problem Definition (1)

➢ A last mile region: $L \times W$ region connected to a hub via line haul of length $J$

➢ Demand density: spatially uniformly distributed and temporally as mean-reverting process.

➢ The fixed-route conventional mode subdivided into $N_c$ routes of width $r$ and length $L$

➢ The flexible service mode subdivided into a grid of $N_f$ zones of area $A$.

Kim & Schonfeld (2014)
Problem Definition (2)

➢ Given:
  ➢ Parameters of a stochastic process for demand density (as mean-reverting) fitted to historical data
  ➢ Current demand density
  ➢ Current operating state (fixed vs flexible, or 1-veh flexible vs 2-veh flexible)

➢ Determine whether or not to switch operating state at current time to minimize operating cost

Kim & Schonfeld (2014)
“Market entry-exit” switching option with OU process

- Two operating modes (e.g. “in market” vs “out of market”)

- Both modes governed by single OU stochastic process, e.g. demand $Q(t)$

- Each mode’s incremental cost or payoff function at $t$: $C_0(Q(t)), C_1(Q(t))$ where a single threshold $Q^*$ exists for deterministically choosing one mode over the other

- Optimal policy under infinite horizon determines threshold $Q_L$ and $Q_H$ to optimize value function (current and future expected payoffs)
SWITCHING BETWEEN FIXED ROUTE (MODE 1) AND ON-DEMAND (MODE 0)
For fixed route transit, $C_1 =$ the sum of the bus operating cost $C_o$, user in-vehicle cost $C_v$, user waiting cost $C_w$, and user access cost $C_x$.

$$C_1(Q) = C_o + C_v + C_w + C_x = a_1 + b_1 Q + d_1 Q + e_1 Q$$

$C_0 = $ sum of bus operating cost $C_o$, user in-vehicle cost $C_v$, user waiting cost $C_w$, and user access cost $C_x$

$$C_0(Q) = a_0 Q^\frac{4}{5} + b_0 Q^\frac{2}{3} + d_0 Q$$

where $a, b, d, e$ are functions of region size, fleet size, and operating speeds – route spacing $r$ (for fixed route), zone size $A$ (for flexible), and vehicle size $S_c$ are endogenously determined to minimize cost (from Chang and Schonfeld, 1991a)
The immediate cost savings accrued from time $t$ to $t + dt$ when switching from flexible to fixed route mode is:

$$\Phi(Q(t)) = C_0(Q(t); S_f) - C_1(Q(t); S_c)$$

If $\Phi > 0$, there is cost savings switching from flexible to fixed route mode.
If $\Phi < 0$, there is cost savings switching from fixed route to flexible service.
Policy valuation based on asset equilibrium pricing

- The option value of using flexible bus operating mode $V_0(Q)$

$$\frac{1}{2} \sigma^2 Q^2 V''_0(Q) + \mu (m - Q) V'_0(Q) - \rho V_0(Q) = 0$$

- The option value of using conventional bus operating mode $V_1(Q)$

$$\frac{1}{2} \sigma^2 Q^2 V''_1(Q) + \mu (m - Q) V'_1(Q) - \rho V_1(Q) + \Phi(Q) = 0$$
Asset equilibrium conditions

- Define $Q_L$ as demand threshold to switch from fixed to flexible, and $Q_H$ from flexible to fixed

When $F^+ = F^- = 0$, $Q_H = Q_L$

- At asset equilibrium, value matching between two modes

\[ V_0(Q_H) = V_1(Q_H) - F^+ \]
\[ V_1(Q_L) = V_0(Q_L) - F^- \]

- Smooth pasting

\[ V_0'(Q_H) = V_1'(Q_H) \]
\[ V_0'(Q_L) = V_1'(Q_L) \]

$F^+$ is the cost assumed for switching from flexible bus service to conventional bus service

$F^-$ is the cost of switching from conventional bus service to flexible bus service;
Asset equilibrium conditions

- General solution of $V_0(Q)$ and $V_1(Q)$

$$V_0(Q) = \left[ A_0 H(-\gamma_0, w_0, x) + B_0 \left( \frac{2\mu m}{\sigma^2 Q} \right)^{1-w_0} H(1 - \gamma_0 - w_0, 2 - w_0, x) \right] Q^{\gamma_0}$$

$$V_1(Q) = \left[ A_1 H(-\gamma_1, w_1, x) + B_1 \left( \frac{2\mu m}{\sigma^2 Q} \right)^{1-w_1} H(1 - \gamma_1 - w_1, 2 - w_1, x) \right] Q^{\gamma_1}$$

$$+ E_t \left[ \int_t^\infty \Phi(Q(s)) e^{-\rho(s-t)} ds \right]$$

$H(\cdot)$ is a confluent hypergeometric ("Kummer") function

$$H(\gamma, w, x) = 1 + \frac{\gamma}{w} x + \frac{\gamma(\gamma + 1)x^2}{w(w + 1)2!} + \frac{\gamma(\gamma + 1)(\gamma + 2)x^3}{w(w + 1)(w + 2)3!} + \cdots$$
Asset equilibrium conditions

\[ X = [Q_H, Q_L, A_0, A_1]' \] is uniquely determined by solving

\[
F(X) = \begin{bmatrix}
A_0H_0(Q_H)Q_H^{Y_0} + (\Delta_1A_0 - A_1)H_1(Q_H)Q_H^{Y_1} - E_t \left[ \int_t^\infty \Phi(Q(s)|Q(t) = Q_H)e^{-\rho(s-t)}ds \right] + F^+ \\
A_0H_0(Q_L)Q_L^{Y_0} + (\Delta_1A_0 - A_1)H_1(Q_L)Q_L^{Y_1} - E_t \left[ \int_t^\infty \Phi(Q(s)|Q(t) = Q_H)e^{-\rho(s-t)}ds \right] - F^- \\
A_0M_0(Q_H)Q_H^{Y_0} + (\Delta_1A_0 - A_1)M_1(Q_H)Q_H^{Y_1} + \frac{\partial E_t[\int_t^\infty \Phi(Q(s)|Q(t) = Q_H)e^{-\rho(s-t)}ds]}{\partial Q} \\
A_0M_0(Q_L)Q_L^{Y_0} + (\Delta_1A_0 - A_1)M_1(Q_L)Q_L^{Y_1} + \frac{\partial E_t[\int_t^\infty \Phi(Q(s)|Q(t) = Q_H)e^{-\rho(s-t)}ds]}{\partial Q}
\end{bmatrix}
\]

Due to the complexity of the equations, we obtain the solution numerically.
Model properties

- Sensitivity of switching policy to transportation system parameters

<table>
<thead>
<tr>
<th>Solutions</th>
<th>( Q(0) = 32 \text{ trips/mile}^2/\text{hr} ), operating as flexible service initially</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Transit</td>
<td></td>
</tr>
<tr>
<td>Headway, ( h_c )</td>
<td>0.42</td>
</tr>
<tr>
<td>Vehicle size, ( S_c )</td>
<td>75</td>
</tr>
<tr>
<td>Fleet size, ( F_c )</td>
<td>5</td>
</tr>
<tr>
<td>Route spacing, ( r )</td>
<td>1.41</td>
</tr>
<tr>
<td>Total cost, ( C_{sc} )</td>
<td>2881.1</td>
</tr>
<tr>
<td>Flexible Transit</td>
<td></td>
</tr>
<tr>
<td>Headway, ( h_f )</td>
<td>0.08</td>
</tr>
<tr>
<td>Vehicle size, ( S_f )</td>
<td>7</td>
</tr>
<tr>
<td>Fleet size, ( F_f )</td>
<td>58</td>
</tr>
<tr>
<td>Service zone, ( A )</td>
<td>3.02</td>
</tr>
<tr>
<td>Total cost, ( C_{sf} )</td>
<td>2883.0</td>
</tr>
</tbody>
</table>

\[
\Phi(Q) = 1.9
\]

\[
E_t \left[ \int_t^\infty \Phi(Q)e^{-\rho(s-t)}ds \right] = -39.4
\]

\[
V_0(Q(0)) = 23.5
\]

\[
V_1(Q(0)) = 17.4
\]

\[
Q_L = 28.7
\]

\[
Q_H = 41.8
\]

Indifference band \((Q_H - Q_L)\) = 13.1
Illustration of policy in action
Verification of policy

Experimental design

For the same simulated demand density trajectory of 96 observations, outcome decisions (and accumulated costs) are made for the following policies:

- **Perfect information (oracle) scenario**: determine the optimal switching points deterministically as if the operator knew the demand outcome beforehand;

- **Myopic policy scenario**: determine the optimal switching points whenever the incremental cost threshold ($\Phi=0$) is crossed;

- **Proposed policy scenario based on market entry-exit switching option**: switch to fixed transit whenever $Q(t) > Q_H$ or switch to flexible transit whenever $Q(t) < Q_L$. 
**Experimental design**

\[ Q_{n+1} = Q_n + \mu(m - Q_n)\Delta t + \sigma Q_n \Delta w_n \]

- The performance of the proposed policy

\[ \varpi(\pi) = \frac{R_{my} - \pi}{R_{my} - R_{ph}} \]

- \( R_{ph} \) perfect information scenario
- \( R_{my} \) myopic scenario

Fixed vehicle sizes \( S_f = 8 \), \( S_c = 80 \) seats/vehicle

Average demand density of 40 trips/mile^2/hr
Experimental Results

➢ When switching cost is 0, there is just one threshold $Q^* = 35.6$.

➢ When there is a switching cost $F^+ = F^- = 10$, then the optimal thresholds are $Q_L = 28.7$ $Q_H = 41.8$.

![Graph showing flexible and fixed transit policy changes over time with thresholds $Q_{L}$, $Q_{H}$, and $Q^*$ highlighted.]
Experimental Results

Fig. 8. (a) Myopic switching policy, and (b) perfect information switching policy.

<table>
<thead>
<tr>
<th></th>
<th>Perfect information</th>
<th>Proposed policy</th>
<th>Myopic policy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total discounted cost</strong></td>
<td>46093.77</td>
<td>46150.62</td>
<td>46293.73</td>
</tr>
<tr>
<td>$\omega(\pi)$</td>
<td>1.0000</td>
<td>0.7157</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

\[
\omega(\pi) = \frac{R_{my} - \pi}{R_{my} - R_{ph}}
\]

The proposed policy can reduce the excess cost by **72%** relative to myopic policy.
Model properties

- Switching policy sensitivity to demand density

Fig. 4. (a) Incremental operational cost from fixed to flexible transit
(b) option value of conventional and flexible bus service with respect to demand density.
Model variation: modular vehicle size

This general structure can manage autonomous fleets such as the Next Future Mobility system

- Two different vehicle sizes $S_1$ and $S_2, S_2 = 2S_1$
- $C_{sf1}(Q; S_1) =$ the cost of operating flexible service with vehicle size $S_1$
- $C_{sf2}(Q; S_2)$ for operating flexible service with vehicle size $S_2 = 2S_1$.
- The incremental cost savings function variation $\Phi_v$
  \[
  \Phi_v(Q(t)) = C^*_{sf1}(Q(t); S_1) - C^*_{sf2}(Q(t); S_2)
  \]
Application to vehicle modularity

The experiment is designed to compute the option premium for the added flexibility to switch between two vehicle sizes: $S_1 = 10$ and $S_2 = 20$.

- **Scenario 1**: flexible service with only one fixed vehicle size $S_0$ (static policy),
- **Scenario 2**: flexible service with two vehicle sizes in which the proposed policy is used to determine optimal switching, assuming the system initiates at $S_1$ and having symmetric switching costs $F_S = 10$. 
Application to vehicle modularity

- $F_S = 10$
- $Q_H = 33.5, S_1 \rightarrow S_2$
- $Q_L = 22.8, S_2 \rightarrow S_1$
- $F_S = 0$
- $Q^* = 28.2.$

Fig 10. Proposed switching policy for vehicle modularity.

<table>
<thead>
<tr>
<th></th>
<th>Proposed policy</th>
<th>Static policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative total cost ($)</td>
<td>45946.64</td>
<td>46320.09</td>
</tr>
</tbody>
</table>

The flexibility to switch vehicle size in this case leads to an improvement over a static policy of $373.45 over the 24 hrs.
Conclusion

➢ Optimize dynamic switching of transit service as a market entry-exit real options model with mean-reverting demand density;
➢ Model features a quantifiable hysteresis effect;
➢ Relative to a myopic policy, the performance of the proposed policy can eliminate up to 72% of the excess cost;
➢ An option premium exists for having the flexibility to switch between two vehicle sizes.
Further research

➢ Also applicable to selecting/staging different transportation technologies (LRT vs heavy rail, electric vehicle infrastructure, autonomous vehicle infrastructure, etc.)

➢ Adding significant switching duration (e.g. Li et al, 2015) and decision-dependent stochastic process

➢ An empirical study on a transit operation with real data
Funding support
- Guangdong Provincial Natural Science Foundation (2016A030310223)
- National Science Foundation for Young Scientists of China (71601192)
- China Postdoctoral Science Foundation (2017T100655)
- National Science Foundation (CMMI-1634973)

Citation

Code and data
https://github.com/BUILTNYU/Optimal-Switching

Questions?
Qian-wen Guo: guoqw3@mail.sysu.edu.cn
Joseph Y.J. Chow*: joseph.chow@nyu.edu
Paul Schonfeld: pschon@umd.edu