Supplemental Materials for Coercive Leadership

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This supplement addresses four things. First, we present a few supplemental results that were used in the main paper. Second, we consider the style of information provision in our model (Appendix B). Third, we show that our results are robust to the case where agents imperfectly observe the leader’s policy (Appendix C). Fourth, we consider the leader’s decision to alter the coercive cost (Appendix D).

A Supplemental Results

We begin by deriving the leader’s indirect utility.

Lemma A.1 At any linear strategy \(k\), the leader’s payoff evaluated at the leader’s best-response \(\gamma^*(s_L, b_L, Q, k)\) is

\[-\int_0^1 (\theta + b_L - a_i)^2 \, di,\]

Proof: Recall that the leader’s payoff is

\[-\int_0^1 (\theta + b_L - a_i)^2 \, di,\]

which evaluated at a linear strategy can be written

\[-\int_0^1 (\theta + b_L - (k_0 s_i + k_1 \gamma + k_2 Q + k_3 h_i))^2 \, di.\]

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From (8) by substitution for the leader’s best-response, her payoff in equilibrium can be written as

$$-\int_0^1 \left( \theta + b_L - (k_0 s_i + k_1 \left( \frac{b_L}{k_1} + \frac{1 - k_0}{k_1} \theta - \frac{k_2}{k_1} Q \right) + k_2 Q + k_3 h_i) \right)^2 di,$$

which after canceling terms is

$$-\int_0^1 (k_0 (\theta - s_i) - k_3 h_i)^2 di,$$

and then factoring terms is

$$-k_0^2 \int_0^1 (\theta - s_i)^2 di - k_3^2 \int_0^1 h_i^2 di = -\frac{k_0^2}{\tau_A} - \frac{k_3^2}{\lambda_A}.$$

We next give a more detailed proof of Lemma 2:

Denote the “inattentive” equilibrium when agents do not use information from the leader’s policy by $\tilde{k}$. Since $\alpha_2(\tilde{k}_0) = 0$ then $\alpha_1(\tilde{k}_0) = \frac{\tau_A}{\tau_A + \tau_Q}$, and by (19):

$$\tilde{k}_0 = \frac{\tau_A}{\tau_A + \tau_Q} \frac{\tau_Q}{1 + c + \rho(\tau_A + \tau_Q)}$$

$$= \frac{\tau_A}{(1 + c)\tau_A + (1 + c + \rho)\tau_Q}.$$  

The coefficients $\tilde{k}_1 = \frac{c}{1+c}$ and $\tilde{k}_3 = \frac{1}{1+c+c}$ follow by inspection from (21) and (15) respectively. By substitution in (22),

$$k_2(\tilde{k}_0) = \frac{(\rho\tilde{k}_0 + 1)\tau_Q}{\tau_A + \tau_Q} \frac{\tau_Q}{1 + c}$$

$$= \frac{(\rho\tilde{k}_0 + 1)\tau_Q}{(1 + c)(\tau_A + \tau_Q)}.$$  

**Lemma A.2** In any equilibrium in which $k_1 = 0$, the leader’s policy choice is uninformative.
**Proof:** From (7) we showed that if \( k_1 = 0 \), then any choice is a best response for the leader. Suppose further that the leader’s choice when \( k_1 = 0 \) is informative regarding \( \theta \). Since the agent’s optimal action choice, which by (6), is linear in an agent’s posterior of \( \theta \). Thus, any equilibrium must incorporate the information that can be gained from the leader’s policy into an agent’s posterior of \( \theta \), and thus \( k_1 \neq 0 \), contradicting the supposition that \( k_1 = 0 \). ■

**Lemma A.3** For any linear strategy \( k \),

\[
\mathbb{E} \left[ \int_0^1 (\theta - a_j(k))^2 \, dj \right] = \mathbb{E}[(\theta - A(\theta \mid k))^2] + \mathbb{E} \left[ \int_0^1 (A(\theta \mid k) - a_j(k))^2 \, dj \right]
\]

and

\[
\mathbb{E} \left[ \int_0^1 (A(\theta \mid k) - a_j(k))^2 \, dj \right] = \frac{k_0^2}{\tau_A} + \frac{k_3^2}{\lambda_A}
\]

**Proof:** By computation,

\[
\mathbb{E} \left[ \int_0^1 (\theta - a_j(k))^2 \, dj \right] = \mathbb{E} \left[ \int_0^1 (\theta - A(\theta \mid k) + A(\theta \mid k) - a_j(k))^2 \, dj \right]
\]

\[
= \mathbb{E}[(\theta - A(\theta \mid k))^2] + \mathbb{E} \left[ \int_0^1 (A(\theta \mid k) - a_j(k))^2 \, dj \right] + 2\mathbb{E} \left[ \int_0^1 (a_j(k) - A(\theta \mid k))(A(\theta \mid k) - \theta) \, dj \right].
\]

Since

\[
2\mathbb{E} \left[ \int_0^1 (a_j(k) - A(\theta \mid k))(A(\theta \mid k) - \theta) \, dj \right] = 2\mathbb{E} \left[ A(\theta \mid k) \int_0^1 a_j(k) \, dj - A(\theta \mid k)^2 - \theta \int_0^1 a_j(k) + A(\theta \mid k) \theta \right]
\]

\[
2\mathbb{E} \left[ A(\theta \mid k)^2 - A(\theta \mid k)^2 - A(\theta \mid k)\theta + A(\theta \mid k)\theta \right] = 0
\]
establishing the first part. For the second part, by computation:

\[
\mathbb{E} \left[ \int_0^1 (A(\theta | k) - a_j(k))^2 \, dj \right] = \mathbb{E} \left[ \int_0^1 (k_0 \theta + k_1 \gamma + k_2 Q - (k_0 s_j + k_1 \gamma + k_2 Q + k_3 h_j))^2 \, dj \right]
\]

\[
= \mathbb{E} \left[ \int_0^1 (k_0 \theta - (k_0 s_j + k_3 h_j))^2 \, dj \right] = \mathbb{E} \left[ \int_0^1 (k_0 (\theta - s_j))^2 \, dj \right] + \frac{k_3^2}{\lambda A}
\]

\[
= \frac{k_0^2}{\tau A} + \frac{k_3^2}{\lambda A}
\]

\[\blacksquare\]

## B A Note on Information

In this section we remark on the nature of information provision in our model. To facilitate a comparison, recall the benchmark of our model in which agents are "inattentive" to the fact that the policy chosen by the leader contains information regarding the underlying state of the world \( \theta \).

**Remark 1** In the benchmark where agents are inattentive to the informational content of policy, the leader’s equilibrium policy choice is still informative of \( \theta \).

**Proof:** Recall the precision of the informational content of policy, (10), and substitute \( \tilde{k}_0 \):

\[
\tau_\gamma(\tilde{k}_0) = (1 - \tilde{k}_0)^2 \lambda L,
\]

which establishes the remark. \(\blacksquare\)

The leader’s policy choice in our model contains information about the state of the world even when no one is paying attention to this fact. This contrasts our model with other models of leadership where leaders are thought of as "cheap-talk" senders who do not exercise the kind of coercive influence that we highlight (e.g., Gilligan and Krehbiel 1987; Epstein and O’Halloran 1997; Gailmard and Patty 2012). In this sense, our model more closely resembles a costly signaling environment rather than a setting where signals are
cheap talk messages. Although the leader in our main model is cognizant of the fact that agents use her policy to update their beliefs, she is driven by motives other than the provision of information.

Consider an alternative version of our model in which the leader does not care about the degree to which agents are coordinated, but rather, only cares about the average action of the set of agents. Formally, this means that the leader’s payoff function, in contrast to (1) in the main model, would be

\[- \left( \theta + b_L - \int_0^1 a_i di \right)^2 = - (\theta + b_L - A)^2.\]

First, by direct calculation, the best-response function of this modified leader’s objective is identical to that in the main analysis: the leader in this case chooses the same policy, and consequently, is just as informative to agents as in our main analysis.

**Remark 2** The informational content of policy is higher when agents are attentive, but is independent of whether the leader cares whether agents are attentive.

**Proof:** Recall the precision of the leader’s policy choice from (10), and then it follows that

\[0 < \tilde{\tau}_A = (1 - \tilde{k}_0)^2 \lambda_L < (1 - k_0^*)^2 \lambda_L = \tau_A(k_0^*).\]

Recall from Lemma A.1 that the leader’s payoff at equilibrium is

\[- \frac{(k_0^*)^2}{\tau_A} - \frac{(k_3^*)^2}{\lambda_A}.\]

Notice that \(k_3^* = \tilde{k}_3\), and so determining when the leader’s payoff at equilibrium is higher when agents use the information from the leader’s policy choice reduces to determining when \(\tilde{k}_0 > k_0^*\), which is always true.

For the final part of the remark, we consider an alternative model in which the leader’s
payoff function is given by

\[-(\theta + b_L - \int_0^1 a_idi)^2 = -(\theta + b_L - A)^2.\]

We need to establish two things: (1) this is strategically equivalent to our model, and (2) the leader’s payoff at equilibrium is independent of how agents use the informational content of policy. For the first part, observe that the leader’s problem is

\[
\max_{\gamma} - (\theta + b_L - \int_0^1 (k_0s_i + k_1\gamma + k_2Q + k_3h_i)di)^2.
\]

The first-order condition associated with this leader’s problem is given by

\[
(\theta + b_L - k_0\theta - k_1\gamma - k_2Q) \cdot k_1 = 0.
\]

which is identical to (7). For the second part, the argument is similar to that for Lemma A.1:

\[
-(\theta + b_L - \int_0^1 (k_0s_i + k_1\gamma + k_2Q + k_3h_i)di)^2
= -(\theta + b_L - (k_0\theta + k_1 \left(\frac{k_0}{k_1} + \frac{1-k_0}{k_1}\theta - \frac{k_2}{k_1}Q\right) + k_2Q))^2
= 0,
\]

which is clearly independent of whether \(k_0 = k_0^*\) or \(k_0 = \tilde{k}_0\). ■

This remark highlights two key features of our main model. First, the leader in the main model enjoys the information provided by her policy choice, and second, that this results from her preference for agents to be coordinated around her preferred average action. The modified model is, thus, strategically equivalent to the main model, and as in the main model, the modified leader always gets her most preferred average action (cf. Proposition 3). The leader in this modified model, however, has no preference over whether agents are attentive to the information available in her policy choice. Given the strategic
equivalence between these two models, the change in the informational content of policy resulting from the attentiveness of agents is independent of whether the leader has preferences over the informativeness of her policy choice.

The nature of informativeness in the leader’s policy choice is, thus, distinct from the way in which information is provided in standard signaling models, where the informativeness of actions is typically a function of the sender’s desire to reveal. In our model, the leader may benefit from information revelation, and her policy choice reflects the strategic effects of the informational content of policy, but the revelation of information is not what motivates her policy choice. It is simply that the leader’s policy choice reflects the information that is available to her and this has equilibrium effects on everyone’s choices.

C Imperfect Observability

In this supplement we show that our results are robust to the inclusion of noise in the leader’s policy choice. This extension is important since it provides a closer connection to Dewan and Myatt (2008, 2012), where noise in the leader’s pronouncement reflects her clarity of communication. It is important to observe that idiosyncracies in how individual agents perceive the leader’s policy choice can manifest in two substantively distinct ways. We show that our results hold in both cases. Consider first,

Case 1: Agents’ actions are subject to the idiosyncratic policy, meaning that an individual agent incurs a cost to deviating from her individualized policy;

Case 2: Agents’ actions are subject to a common policy that they do not observe, meaning that every agent incurs a cost from deviating from a common policy, and agents observe idiosyncratic signals of this common policy.

Formally, we capture these cases as follows. In the first case, the leader chooses a policy, γ, and agent \( i \) receives an individualized policy \( g_i = \gamma + \epsilon_i \) where \( \epsilon_i \) is independently drawn from a normal distribution with mean zero and precision \( \tau_g \). Each individual agent pays a
cost for deviations from her idiosyncratic policy. Specifically, the agent’s utility function in this case is
\[
u(a, A, h, \gamma, \theta) = -\rho (L(a) - L) - (a - \theta - h)^2 - c(g_i - a)^2.
\]

In the second case, the leader chooses a policy, \(\gamma\), and agent \(i\) receives a signal of the leader’s policy choice, \(g_i = \gamma + \epsilon_i\), where \(\epsilon_i\) is independently draw from a normal distribution with mean zero and precision \(\tau_g\). Each individual agent pays a cost for deviations from the leader’s policy \(\gamma\). In this case the utility function is as in the main model,
\[
u(a, A, h, \gamma, \theta) = -\rho (L(a) - L) - (a - \theta - h)^2 - c(\gamma - a)^2.
\]
The only difference between this case and the main analysis is that the leader’s policy is imperfectly observed.

In each case an agent solves the optimization problem:
\[
\max_{a_i} \mathbb{E}[-\rho (L(a_i) - L) - (a_i - \theta - h_i)^2 - c(\gamma^g - a_i)^2 | s_i, \gamma_i, Q]. \tag{C.1}
\]
where \(\gamma^g = g_i\) in case 1 and \(\gamma^g = \gamma\) in case 2. In the first case, we take expectations over \(\theta\) and \(A\), whereas in the second case, we take expectations over \(\theta\), \(A\), and \(\gamma\). In either case, the first-order condition associated with (C.1) is
\[
\mathbb{E}_i[\rho(A - a_i) - (a_i - \theta - h_i) + c(\gamma^g - a_i)] = 0,
\]
where \(\mathbb{E}_i\) represents the expectation given \(i\)’s information set. Carrying through expectations, rearranging, and solving for the agent’s optimal action yields:
\[
a_i^*(s_i, h_i, \gamma, Q) = \frac{\rho}{1 + \rho + c} \mathbb{E}_i[A] + \frac{1}{1 + \rho + c} \mathbb{E}_i[\theta] + \frac{c}{1 + \rho + c} \mathbb{E}_i[\gamma^g] + \frac{1}{1 + \rho + c} h_i.
\]
The leader’s sequentially rational optimization problem can be written as

$$\max_{\gamma} - \int_0^1 (\theta + b_L - a_i^*(s_i, h_i, \gamma, Q))^2 di.$$ 

When agent strategies are linear, characterized by vector $k = (k_0, k_1, k_2, k_3)$, then, by substitution, the leader’s problem becomes

$$\max_{\gamma} - \int_0^1 (\theta + b_L - (k_0 s_i + k_1 (\gamma + \epsilon_i) + k_2 Q + k_3 h_i))^2 di.$$ 

which can be written as

$$\max_{\gamma} - \int_0^1 (\theta + b_L - (k_0 s_i + k_1 \gamma + k_2 Q + k_3 h_i))^2 di - \frac{k_2^2}{\tau_g}.$$ 

Since $\frac{k_2^2}{\tau_g}$ is a constant for the leader, her policy choice is the same as in the main model. The precision of the informational content in the leader’s policy is

$$\tau_\gamma(k_0) = (1 - k_0)^2 \lambda_L, \quad (C.2)$$

and straightforward calculations show that the precision of $\gamma_i$ is

$$\tau_{\gamma_i}(k_0) = \frac{(1 - k_0)^2 \lambda_L \tau_g}{\tau_g + (1 - k_0)^2 \lambda_L}.$$ 

The remainder of the analysis follows by substitution of $\tau_{\gamma_i}^g$ for $\tau_\gamma$. 

D Demanding Compliance

The leader in our model prefers to coordinate all political actors around her most preferred outcome. The magnitude of the coercive cost, $c$, provides a measure of the leader’s ability to motivate compliance among political actors. As $c$ increases, actions that deviate from the leader’s policy choice, $\gamma$, become harder for individuals to endure. Thus, increasing $c$
also increases the overall level of compliance with the leader’s policy choice. Consequently, the leader strictly prefers higher coercive costs to lower coercive costs.

To consider how the leader might choose a level of compliance among political actors, we allow the leader to have an ex ante choice of the non-compliance cost \( c \). Formally, suppose there is an initial non-compliance cost given by \( c_0 > 0 \), and suppose that it costs the leader to change the non-compliance cost according to the quadratic loss function, \( \kappa(c - c_0)^2 \), where \( \kappa > 0 \). To summarize, we are considering an extension in which the leader’s payoff is

\[
- \int_0^1 (\theta + b_L - a_i)^2 \, di - \kappa(c - c_0)^2.
\]

The leader’s payoff reflects that she must exert effort or resources in expanding (or even reducing) her ability to coerce her followers.

**Remark 3** There exists an interior choice of coercive costs, \( c^\dagger(\kappa, c_0, \rho, \tau_A, \lambda_A, \lambda_L; k) \), and it is independent of the state of the world \( \theta \) as well as the leader’s bias \( b_L \).

**Proof:** From Proposition 18, we know that for any linear equilibrium \( k^* \), \( A(\theta \mid k^*) = \theta + b_L \). Combining this with the expression from Lemma A.1 the leader’s objective function can be written as

\[
\max_c - \frac{(k_0^*)^2}{\tau_A} - \frac{(k_3^*)^2}{\lambda_A} - \kappa(c - c_0)^2
\]

The first order condition associated with this problem is

\[
\frac{dk_0^*}{dc} (c^\dagger) \cdot \frac{k_0^*(c^\dagger)}{\tau_A} - \frac{1}{\lambda_A(1 + \rho + c^\dagger)^3} = \kappa(c^\dagger - c_0),
\]

which can be rewritten as

\[
\frac{\alpha_1(k_0^*)}{\tau_A(1 + c^\dagger + \rho(1 - \alpha_1(k_0^*))^3(\Omega'(k_0^*) \cdot \frac{d\alpha_1}{dk_0^*} - 1))} = \frac{1}{\lambda_A(1 + \rho + c^\dagger)^3} = \kappa(c^\dagger - c_0).
\]
The value of coercive cost that the leader chooses, $c^l(\kappa, c_0, \rho, \tau_A, \lambda_A, \lambda_L; k)$ depends on many things, and moreover, its functional form can be quite complicated. However, one important feature of the leader’s choice in coercive cost is that it is independent of the state of the world, $\theta$, as well as the leader’s private bias, $b_L$. This fact implies that even if the leader can choose her level of coercive cost, such a choice would not be used by the leader to reveal information, and hence, cannot act as a secondary channel by which the leader might attempt to “signal” to agents. The logic behind this stark result is that all leaders, regardless of the state of the world and their private bias, want to choose a higher coercive cost.

**References**


