Abstract

We examine the intuition that in supermajoritarian settings, polarization and policymaking gridlock are fundamentally linked, but that a pressing common problem can reduce both. When actors’ individual costs from a policy addressing such a problem differ, their preferences over the appropriate policy respond asymmetrically to increases in the magnitude of the problem. In a broad range of circumstances such increases can give rise to increased polarization, but may also simultaneously yield net welfare-enhancing policy adjustments rather than entrenchment of gridlock. The association of polarization and gridlock is contingent on two underlying factors: how the problem responds to the policy solution, and the location of the status quo policy when the extent of the problem changes. We illustrate the model’s logic by comparing U.S. national policy making in the Progressive Era and the present.
Introduction

Contemporary accounts of political dysfunction (e.g., Binder 2003; Mann and Ornstein 2012) emphasize despair over the policy consequences of political polarization in the U.S.: mired in policy gridlock, a polarized system transforms every issue into a partisan battle. Perhaps, under such circumstances, a pressing common problem could push competing interests to set aside partisan rancor and focus instead on mutually beneficial solutions – a “rally effect.” In this note, we provide a simple formal account of collective decision-making that suggests there may be good reason to doubt both the rationale for the despair and for the hope.

In our model, actors in a political system (e.g., citizens in a polity, states’ representatives in a federal legislature, etc.) experience a common harm. A policy can mitigate the harm, but the actors bear its costs asymmetrically. A helpful example is pollution: all citizens may be harmed by sulfur-dioxide, but citizens in coal-producing regions would disproportionately bear the economic consequences of emissions controls and so differ in their preferred levels of regulation. An increase in the magnitude of harm will increase everyone’s preferred scope of remedial policy, but at different rates. In a broad, behaviorally plausible array of circumstances, the rate of increase will be higher for those that prefer more regulation. A similar effect will operate when the harm is from actions by an unruly foreign power and politicians (or allies) all dislike its actions, but perceive the costs of a robust response differently.

We develop several implications with respect to such settings. First, an increase in the common problem can produce greater polarization rather than greater consensus. Second, such increases in polarization may yet be accompanied by policy innovation rather than entrenched gridlock\(^1\) even under supermajoritarian institutions. Third, in such settings, the net effect of an increase in the collective harm may be a utilitarian welfare improvement. We show how the occurrence of these effects depends on the following three factors: the

\(^{1}\text{It is straightforward that polarization and policy innovation can jointly occur in canonical gridlock models (Brady and Volden 1998; Krehbiel 1998) given exogenous electoral shocks.}\)
relationship between the policy solution and the experienced harm, the location of the status quo policy before the increase in the harm, and the institutional structure of policy making. Our analysis suggests that preference polarization is not by itself a sufficient statistic for political dysfunction, and must be placed in its proper policy-making and institutional context. We illustrate the implications of this idea with a comparison of U.S. national policy making in the Progressive Era and the present.

The Model

Suppose a continuum of actors, indexed by $i$, with measure one. Each actor suffers a common problem, e.g., pollution. Let $g(\omega, r)$ represent the harm to an actor associated with a level of the (unremediated) common problem $\omega \in \mathbb{R}_+$ and a remedial policy imposed centrally, $r \in \mathbb{R}_+$, e.g., regulatory abatement. (As a shorthand, we will refer to the policy henceforth as regulation.) We assume that harm is increasing in the magnitude of the problem itself at an increasing rate, and decreasing in the regulatory solution at a decreasing rate; and that the marginal benefit of the regulatory solution is increasing in the magnitude of the unregulated harm.\(^2\) This last assumption is consistent with the conventional wisdom described above, as well as with the historic growth of regulation in the U.S., in which demand for a policy response increased following, e.g., industrialization, increasing medical costs, etc.\(^3\)

There is an actor-specific cost of regulation, $\alpha_i r$, with the cdf of $\alpha_i \in \mathbb{R}_+$ given by $P(\alpha_i)$. An actor’s utility is given by $-g(\omega, r) - \alpha_i r$. To keep the focus on the policy consequences of the commonly experienced harm, we do not explicitly model the production choices of different actors and their consequences for the overall level of harm in a general equilibrium. One can think of the cost heterogeneity as a reduced form for variation in the opportunity cost of regulation.

\(^2\)Formally, $\frac{\partial g}{\partial \omega} > 0$, $\frac{\partial g}{\partial r} < 0$, $\frac{\partial^2 g}{\partial \omega^2} \geq 0$, $\frac{\partial^2 g}{\partial r^2} > 0$, and $\frac{\partial^2 g}{\partial r \partial \omega} < 0$.

\(^3\)For alternative formalizations of regulatory abatement with similar properties, see Gordon and Hafer (2005, 2007). Callander and Martin (2017) study the effects of policy decay that can lead to a welfare loss akin to harm in our model. As will become clear below, the mechanism by which the associated loss induces policy change differs from the one here.
cost of foregone production stemming from a more stringent regulatory policy.\(^4\)

Let \(S \in \mathbf{S}\) denote an institution that maps the preference profile, the common problem, and the status quo policy, denoted \(r^0\), into an equilibrium level of regulation \(r^*\). (Formally, \(S := \mathcal{U} \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+\), where \(\mathcal{U}\) is the set of all preference profiles \(\mathcal{U}(P(\cdot)) \) given \(P(\cdot)\), the distribution of the \(\alpha_i\).) We will assume that \(S\) contains q-rules, as well as legislative bargaining institutions with gatekeepers or veto players such as Pivotal Politics (Krehbiel 1996, 1998) and Negative Agenda Control (Cox and McCubbins 2005). Given single-peaked preferences, these oft-studied institutions, with the exception of simple majority rule, yield a gridlock interval for at least some preference profiles: a (non-degenerate) compact and convex interval of policies that are gridlocked, i.e., cannot be beaten by another policy even if it has majority support. We will focus on this property below.

Our first result (all proofs are in the SI) describes actors’ induced ideal points:

**Lemma 1**  
1. Each actor has single-peaked preferences with ideal regulation \(\hat{r}_i\).

2. For all \(i\), \(\hat{r}_i\) is weakly increasing in \(\omega\) and decreasing in \(\alpha_i\) (and strictly if \(\hat{r}_i > 0\)).

The lemma is intuitive: it states, simply, that each actor has a uniquely defined ideal level of regulation; unsurprisingly, this level is lower for actors that would incur a high cost of implementing the regulation, and higher when the common problem is more serious.

Our first main result concerns ideal point polarization: for any two actors \(i\) and \(j\), polarization is the absolute difference between their ideal levels of policy: \(|\hat{r}_i - \hat{r}_j|\). A common metric in the empirical literature on U.S. politics used as a shorthand for political conflict, for example, is partisan polarization: the difference between the mean (or median) ideal points of Republican and Democratic members of Congress (e.g., McCarty, Poole, and Rosenthal 2006). Let \(Q \equiv \frac{g_{r\omega}(\omega,r)}{g_r(\omega,r)}\) be the ratio of the cross-partial derivative of \(g(\omega,r)\) with

\(^4\)For clarity, we focus on this general reduced form. The main thrust of our results holds under convex costs, heterogeneity of benefit (rather than cost), and, for a subset of actors, the presence of discontinuities in the harm function (as long as single-peakedness is preserved).
respect to $r$ and $\omega$ and the second derivative of $g(\omega, r)$ with respect to $r$. Then:

**Proposition 1** Polarization between any actors $i$ and $j$ with $\hat{r}_i < \hat{r}_j$ is decreasing in the magnitude of the common problem $\omega$ if and only if $Q(\hat{r}_i) < Q(\hat{r}_j)$ and increasing in the magnitude of the common problem $\omega$ if and only if $Q(\hat{r}_i) > Q(\hat{r}_j)$.

At the most basic level, the result says that an increase in the benefit of a collective policy can sometimes exacerbate, and at other times mitigate, political conflict. The key determinant is the shape of the harm function, as summarized by $Q$. To interpret the condition on the shape of the harm function, consider the following example, which imposes additional structure on $Q$. Suppose that the harm from the common problem and benefit from remediation are multiplicatively separable, that is, $g(\omega, r) = f(\omega)h(r)$. Then a sufficient condition for polarization increasing in the magnitude of harm $\omega$ is $\left(\frac{h''(r)}{h'(r)}\right)' < 0$. This is the definition of *decreasing absolute risk aversion* (DARA) in the benefits to regulatory remediation (Arrow 1965). Informally, DARA implies that if one were able to monetize the benefits of remediation, actors experiencing a higher level of regulation (and so less harm from the common problem) would be less risk-averse with respect to the associated benefits. The condition is consistent with numerous functional forms (e.g., $g(\omega, r) = \omega^k \frac{k}{m+r}$ for $k \geq 1$ and $m > 0$), as well as considerable observational and experimental evidence (e.g., Chavas and Holt 1996; Saha, Shumway, and Talpaz 1994).

Because Proposition 1 holds for any two states, it will hold for standard empirical measures of polarization: for example the distance between the median or mean ideal points of different “parties” (corresponding to a partition of $\alpha$ into actors with high and low remediation costs). To state the implication for the full voting body, consider the gridlock interval induced by supermajoritarian institutions, like $q$-rules with $q > 0.5$, or Pivotal Politics. Let $\hat{r}_l$ denote the ideal level of regulation for an actor at the extreme low end of the gridlock interval, and $\hat{r}_h$ the ideal level for an actor at the extreme high end of the gridlock interval. A variety of polarization of particular empirical interest is the *width of the gridlock interval*,

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\( \hat{r}_h - \hat{r}_l \): naturally, the wider the gridlock interval, the more we expect policy to be gridlocked, ceteris paribus. We will say that the gridlock interval expands (contracts) in response to some exogenous stimulus if the width of the gridlock interval increases (decreases). Relatedly, we will say that the gridlock interval shifts rightward (leftward) in response to some exogenous stimulus if and only if both \( \hat{r}_h \) and \( \hat{r}_l \) increase (decrease). The following remark, implied by the conjunction of Lemma 1 and Proposition 1, describes changes to the position and width of the gridlock interval in response to a change in the common problem \( \omega \).

**Remark 1** Suppose \( S \in S \) and \( \mathcal{U}(P(\cdot)) \in \mathcal{U} \) such that there exists a gridlock interval. As the common problem \( \omega \) increases, the gridlock interval shifts rightward and (a) expands if \( Q(\hat{r}_l) > Q(\hat{r}_h) \); (b) contracts if \( Q(\hat{r}_h) > Q(\hat{r}_l) \); or (c) maintains constant width otherwise.

Remark 1 elaborates on Proposition 1 in the context of the gridlock interval: it shows that when an increase in \( \omega \) expands the gridlock interval, that increase happens not by a simultaneous leftward shift in \( \hat{r}_l \) and rightward shift in \( \hat{r}_h \); but rather by a rightward shift in \( \hat{r}_l \) and a larger rightward shift in \( \hat{r}_h \). This is illustrated in Figure 1 (ignoring \( r^o \) for now).

Insofar as it permits policies to remain in effect even when they deviate from the socially optimal one, gridlock is often understood as reflecting political “failure.” Hence, the width of the gridlock interval is taken as an indicator of the likelihood of such failure. However, that width is independent of the location of the status quo, \( r^o \), and as such is not a sufficient statistic of gridlock itself. Our next result describes how the relationship between the width

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5Importantly, the width of the gridlock interval may differ from partisan polarization, although these measures are likely to be correlated. See, e.g., McCarty 2007, 235.
of the gridlock interval and gridlock respond to changes in the common problem. Let \( \hat{r}_i(t) \) denote \( i \)'s ideal policy at time \( t = \{0, 1\} \), where \( t = 0 \) denotes pre-shock and \( t = 1 \) post-shock.

**Proposition 2** Suppose \( S \in \mathbb{S} \) and \( \mathcal{U}(P(\cdot)) \in \mathcal{U} \) such that there exists a gridlock interval and it contains the status quo, \( r^o \in [\hat{r}_l(\omega_0), \hat{r}_h(\omega_0)] \). Then an increase (decrease) in the common problem that expands the gridlock interval will be accompanied by continued gridlock if and only if \( r^o \geq \hat{r}_l(\omega_1) \) \( (r^o \leq \hat{r}_h(\omega_1)) \).

The implication of this result is that changes to the political environment that bring about greater polarization (wider gridlock interval) need not correspond to the persistence of gridlock. In fact, quite the contrary: such changes can facilitate policy corrections. To understand the intuition, suppose first that \( Q(\hat{r}_l(\omega_0)) > Q(\hat{r}_h(\omega_0)) \), so that an increase in the problem yields an expansion and rightward shift in the gridlock interval. If \( r^o \) is sufficiently close to \( \hat{r}_l(\omega_0) \), an increase in \( \omega \) will not only yield an increase in the width of the gridlock interval, but also increase \( \hat{r}_l(\omega_0) \) to the point that \( r^o \) now falls outside of the post-increase interval. The result will be equilibrium policy change in spite of the increase in polarization. Figure 1 displays this logic. A similar intuition holds if \( Q(\hat{r}_h(\omega_0)) > Q(\hat{r}_l(\omega_0)) \), when a decrease in \( \omega \) yields a leftward shift and increase in the width of the gridlock interval. In that case, \( r^o \) sufficiently close to \( \hat{r}_h(\omega_0) \) will fall outside of the post-decrease gridlock interval, yielding policy adjustment. By contrast, if a gridlocked \( r^o \) is sufficiently far from the edges of the gridlock interval, then policy will remain unchanged before and after the change.

What are the welfare implications of an increase in the common problem? Such an increase has both direct and indirect effects on aggregate welfare. The direct effect is negative and accords with common intuition: larger \( \omega \) is worse for everyone. But the indirect effect is potentially positive: if the increase in \( \omega \) disequilibrates a heretofore gridlocked status quo, then the equilibrium adjustment may bring the policy closer to the social optimum than it was before the shock, enhancing aggregate welfare.

**Proposition 3** Holding fixed the preference profile and a status quo policy, an increase in
the magnitude of the common problem can lead to either a net social welfare improvement or net social welfare decline, contingent on the institutional environment.

The intuition behind this result hinges on whether the institution permits relatively small shocks to the problem to be met by large re-equilibrating changes to policy. When they do, the indirect effect can dominate the direct one; otherwise, the net effect will be negative.

Suppose the median actor is also the mean \( (\alpha_m = \overline{\alpha}) \). Under this condition, the median’s ideal point corresponds to the aggregate welfare-maximizing policy. Under simple majority rule, an increase in the common problem will never lead to an improvement in aggregate welfare: equilibrium adjustment to the social optimum will be proportionate to any positive shock to \( \omega \), but will yield a lower level of aggregate utility. In contrast, an institution that can generate disproportionately large equilibrium adjustments is negative agenda control (Cox and McCubbins 2005). In that environment, a pivotal legislator chooses whether to permit proposals to the floor (at which point the proposal is considered under open agenda). In the Appendix, we demonstrate (given a utility function satisfying DARA) that when the status quo policy before the shock is sufficiently close the the left pivot, there exists a nonempty set of positive shocks to \( \omega \) that will lead to a net improvement in aggregate welfare (while simultaneously increasing polarization).\(^6\)

**Polarization and Legislative Innovation in U.S. Politics**

A key implication of our analysis is that the conventional identification of increased polarization with policy gridlock and decreased legislative productivity (e.g., Mann and Ornstein 2012; McCarty, Poole, and Rosenthal 2006, ch. 6; McCarty 2007) is incomplete. Yet, commentators are not wrong in identifying these features as definitive of contemporary politics. Work by McCarty, Poole, and Rosenthal (2006) has demonstrated that partisan polarization in Congress, defined as the ideological distance between the Republican and Democratic

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\(^6\)We also show that under these assumptions, net welfare improvements stemming from positive shocks to \( \omega \) are infeasible under the Pivotal Politics model (Krehbiel 1996, 1998).
party caucuses, is at historically high levels unseen since the Progressive Era a century ago (1900-1916). And in the past two decades, major legislative accomplishments have been few and far between. Those that have passed (e.g., the Affordable Care Act, the Dodd-Frank Financial Reform Act, and the Stimulus Package) did so over the strenuous objections of a unified minority party, a pattern likely to continue in the current political climate.

And yet no observer would confuse our current politics with those of the Progressive Era, despite its similarly high levels of partisan polarization. That era is remembered for enormous legislative productivity and ferment, with landmark legislation that laid the groundwork for the 20th century national administrative state, passing with bipartisan majorities. Why is partisan polarization coincident with an expansion of national governance in one period and stalemate in another? Nominally, congressional institutions made obstructionist tactics easier then than today: before the adoption of Senate Rule 22 in 1917, debate in the senate could conclude only by unanimous consent. However, filibusters are far more common in the contemporary era (Binder and Smith 1996; Wawro and Schickler 2007).

Recall from our formal analysis that actors’ preferred levels of policy are increasing in the extent of commonly experienced problems, and may increase faster for actors for whom the costs of regulation are lower. This yields the positive relationship between extent of the problem and ideal point polarization. The advent of the national market following industrialization in the late 19th Century (Bensel 2000) was accompanied by both a rapid increase in externality-inducing production (e.g., in mining (Gordon and Hafer 2013) and food and drug manufacturing (Young 1989)) and by the increased public awareness of the presence of those externalities and their potentially detrimental effects. Contemporary scholarship gives us the other half of the relationship: a high degree of partisan polarization in Congress at the turn of the 20th century, which, per our model, is consistent with the disruptive changes brought about by industrialization. (To be sure, the formal analysis above holds constant the economic benefits associated with the increase in the common problem, and so accordingly, comparisons of net social welfare before and after industrialization are inadvisable.)
In contrast with today, however, the national government’s footprint at the outset of the Progressive Era was low. Viewed through the prism of our model, the increase in remediable harm from the excesses of industrialization pushed the low end of the gridlock interval upward, to the point where the previously gridlocked low level of status quo national policy was now outside of the gridlock interval, enabling relatively consensual (in most cases) legislative changes to higher levels of regulation. The result was the policy innovation observed during the period. By contrast, in the contemporary period, the status quo federal footprint is relatively high, and increases in contemporary polarization correspond to conservative shifts among Republicans (McCarty et al. 2012), leaving the status quo firmly gridlocked.

References


Gordon, Sanford C., and Catherine Hafer. 2005. “Flexing Muscle: Corporate Political Ex-


A Online Supplemental Appendix For Gordon, Sanford C., and Dimitri Landa, “Common Problems (or, What’s Missing from the Conventional Wisdom on Polarization and Gridlock)”

Proof of Lemma 1

\( i \)'s first order condition is given by

\[ -g_i(\omega, r) - \alpha_i = 0. \] (A.1)

1. Single-peakedness of \( \hat{r}_i \) is guaranteed by global concavity of \(-g(\omega, r)\) in \( r \).

2. Implicitly differentiating (A.1) with respect to \( \alpha_i \) yields

\[ -g_{rr}(\omega, r) \frac{\partial r}{\partial \alpha_i} = 1, \]

or

\[ \frac{\partial r}{\partial \alpha_i} = - (g_{rr}(\omega, r))^{-1} < 0. \]

Implicitly differentiating (A.1) with respect to \( \omega \) yields

\[ -g_{rw}(\omega, r) - g_{rr}(\omega, r) \frac{\partial r}{\partial \omega} = 0, \]

or, simplifying,

\[ \frac{\partial r}{\partial \omega} = - \frac{g_{rw}(\omega, r)}{g_{rr}(\omega, r)} > 0. \] (A.2)
Proof of Proposition 1

Let \( \tilde{r}_i < \tilde{r}_j \), so polarization is \( \tilde{r}_j - \tilde{r}_i \). Polarization is increasing in \( \omega \) if and only if

\[
\frac{\partial \tilde{r}_j}{\partial \omega} - \frac{\partial \tilde{r}_i}{\partial \omega} > 0.
\] (A.3)

Substituting for \( \frac{\partial r}{\partial \omega} \) from (A.2), (A.3) is equivalent to \( Q(\tilde{r}_i) > Q(\tilde{r}_j) \).

Proof of Proposition 2

There are two cases to consider:

1. \( Q(\tilde{r}_l(\omega_0)) > Q(\tilde{r}_l(\omega_0)) \). Then an expansion of the gridlock interval implies \( \omega_1 > \omega_0 \), which itself implies \( \tilde{r}_l(\omega_0) < \tilde{r}_l(\omega_1) \) and \( \tilde{r}_h(\omega_0) < \tilde{r}_h(\omega_1) \). By assumption, \( r^o \leq \tilde{r}_h(\omega_0) \), so \( r^o < \tilde{r}_h(\omega_1) \). Therefore, by the definition of the gridlock interval, gridlock will persist if and only \( r^o \geq \tilde{r}_l(\omega_1) \).

2. \( Q(\tilde{r}_l(\omega_0)) < Q(\tilde{r}_l(\omega_0)) \). Then an expansion of the gridlock interval implies \( \omega_1 < \omega_0 \), which itself implies \( \tilde{r}_l(\omega_1) < \tilde{r}_l(\omega_0) \) and \( \tilde{r}_h(\omega_1) < \tilde{r}_h(\omega_0) \). By assumption, \( r^o \geq \tilde{r}_l(\omega_0) \), so \( r^o > \tilde{r}_l(\omega_1) \). Therefore, by the definition of the gridlock interval, gridlock will persist if and only \( r^o \leq \tilde{r}_h(\omega_1) \).

Proof of Proposition 3

The proof proceeds by describing the different welfare properties of the model under two different institutional structures, which we do in two lemmata. A third lemma, which is not necessary for the proof of the Proposition, describes welfare properties of the model under a third institutional structure, *Pivotal Politics*, discussed in the text.

The three institutional structures are:

**Majority Rule with Open Agenda.** Any legislator can make a proposal, and the proposal that is not opposed by a majority of legislators is adopted.
Negative Agenda Control (Cox and McCubbins 2005). Any legislator can make a proposal, but a pivotal legislator \( c \) (e.g., the majority party caucus median) with preference parameter \( \alpha_c \) chooses whether to permit the proposal to the floor (open the gate). If the gate is opened, any proposal may be considered (the agenda is open) and the proposal that is not opposed by a majority of legislators is adopted. Otherwise, the status quo policy remains in effect.

Pivotal Politics (Krehbiel 1996, 1998). The median legislator sets the agenda by choosing a proposal \( r \) or accepting the exogenous status quo \( r^0 \). The proposal is implemented if it is supported by a majority that includes the filibuster pivot and either the veto pivot or veto override pivot. (By construction, the median legislator’s ideal point lies between these two pivots.) Otherwise, the status quo policy remains in effect.

Assume \( \alpha_m = \overline{\alpha} \), and impose the following functional form for an actor’s utility:

\[
u_i(r) = -\frac{\omega}{1 + r} - \alpha_i r. \tag{A.4}\]

It is straightforward to demonstrate that \( \hat{r}_i = \max\{0, \sqrt{\frac{\omega}{\alpha_i}} - 1\} \), and that aggregate utility \( U(\cdot) \) corresponds to the utility function of the actor with cost parameter \( \overline{\alpha} \). Let \( \hat{r}_m(\omega_t) \) be the legislative median’s ideal policy given harm \( \omega_t \), and denote with \( \tilde{r}_i(r, \omega_t) \) the policy that gives actor \( i \) the same utility as \( r \) given \( \omega_t \) (i.e., \( u_i(\tilde{r}(r, \omega_t)) = u_i(r, \omega_t) \)).

We will consider the consequences of a positive shock in harm \( \Delta \in \mathbb{R}_{++} \) sufficient to push \( \omega \) from \( \omega_0 \) to \( \omega_1 = \omega_0 + \Delta \), and begin with majority rule with an open agenda:

**Lemma A.1 (Shocks to Harm under Majority Rule)** Under majority rule with an open agenda, there exists no \( \Delta \) that yields an improvement in aggregate welfare.

**Proof.** Equilibrium policy is given by \( r^*(\omega_t) = \hat{r}_m(\omega_t) \). Aggregate welfare corresponds to the welfare of the mean voter, which by assumption is equivalent to the welfare of the
median voter. Therefore $r^*(\omega_t)$ is the aggregate welfare maximizing policy. The result then follows immediately from the envelope theorem.

Turning next to negative agenda control, assume that $\alpha_t > \alpha_m$, so $\hat{r}_t(\omega_t) < \hat{r}_m(\omega_t)$.

**Lemma A.2 (Shocks to Harm under Negative Agenda Control)** Let $\alpha_c > \alpha_m$ (such that $\hat{r}_c(\omega_t) < \hat{r}_m(\omega_t)$), and suppose the status quo $r^\circ$ is gridlocked. Under Negative Agenda Control, there exists a nonempty set of shocks increasing the common problem $\omega$ that lead to improvement in aggregate welfare if and only if $r^\circ$ is sufficiently close to, and to the right of, $\tilde{r}_c(\hat{r}_m(\omega_0), \omega_0)$.

**Proof.** Equilibrium policy is given by

$$r^*(\omega_t) = \begin{cases} \begin{array}{ll}
 r^\circ & \text{if } r^\circ \in [\tilde{r}_c(\hat{r}_m(\omega_t), \hat{r}_m(\omega_t))]
 \end{array} \end{cases} \hat{r}_m(\omega_t) \text{ otherwise.}$$  \hspace{1cm} (A.5)

Some algebra reveals

$$\tilde{r}_c(\hat{r}_m, \omega_t) = \frac{\sqrt{\alpha_m \omega_t}}{\alpha_c} - 1.$$ \hspace{1cm} (A.6)

For a welfare enhancing shock to occur, three conditions must hold:

**Condition 1.** Ex ante gridlock. $r^\circ \in [\tilde{r}_c(\hat{r}_m(\omega_0), \omega_0), \hat{r}_m(\omega_0))$]

**Condition 2.** Ex post ungridlocked. $r^\circ < \tilde{r}_c(\hat{r}_m(\omega_1), \omega_1)$.

Substituting $\omega_1 = \omega_0 + \Delta$ into (A.6) and rearranging, this condition is equivalent to

$$\Delta > \frac{\alpha_c^2 A - \alpha_m \omega_0}{\alpha_m} \equiv \Delta,$$  \hspace{1cm} (A.7)

where $A = (1 + r^\circ)^2$.

**Condition 3.** Aggregate welfare enhancement. If $r^*(\omega_1) = \hat{r}_m(\omega_1)$,

$$U(\hat{r}_m(\omega_1), \omega_1) = \alpha_m - 2\sqrt{\alpha_m (\omega_0 + \Delta)}.$$
Given $U(r, \omega_1) = u_m(r, \omega_1)$, Condition 3 may be expressed as

$$-\frac{\omega_0}{1 + r^o} - \alpha_m r^o < \alpha_m - 2\sqrt{\alpha_m(\omega_0 + \Delta)},$$

which simplifies to

$$\Delta < \frac{(\omega_0 - \alpha_mA)^2}{4\alpha_mA} \equiv \overline{\Delta}.$$  

The interval $(\underline{\Delta}, \overline{\Delta})$ is nonempty if and only if $\underline{\Delta} < \overline{\Delta}$. Substituting from above, this condition is equivalent to

$$\frac{\alpha_m^2 A - \alpha_m \omega_0}{\alpha_m} < \frac{(\omega_0 - \alpha_mA)^2}{4\alpha_mA}.$$

This is a quadratic inequality in $A$, satisfied for

$$-\frac{\omega_0}{2\alpha_c + \alpha_m} < A < \frac{\omega_0}{2\alpha_c - \alpha_m}.$$

The first term in this triple inequality is negative, while the third is always positive. From condition 1, $A > \frac{\alpha_m \omega_0}{\alpha_c^2}$. Comparing this lower bound on $A$ with the upper bound from the above, the latter exceeds the former if and only if $\alpha_c > \alpha_m$. By assumption, $\hat{r}_l(\omega_t) < \hat{r}_m(\omega_t)$, implying $\alpha_c > \alpha_m$. Turning to the second part of the triple inequality, substituting for $A$, and solving yields

$$r^o < \sqrt{\frac{\omega_0}{2\alpha_c - \alpha_m}} - 1,$$

which denotes the highest status quo for which a welfare-enhancing shock is feasible. ■

**Lemma A.3 (Shocks to Harm under Pivotal Politics)** Suppose the status quo $r^o$ is gridlocked. Under Pivotal Politics, there exists no $\Delta$ that yields an improvement in aggregate welfare.

**Proof.** Assume without loss of generality that the filibuster pivot has ideal point $\hat{r}_l(\omega_t)$ less
than that of the veto pivot \( \hat{r}_h(\omega_t) \). Equilibrium policy is given by

\[
    r^*(\omega_t) = \begin{cases} 
    \min \{ \tilde{r}_l(r^o, \omega_t), \tilde{r}_m(\omega_t) \} & \text{if } r^o < \hat{r}_l(\omega_t) \\
    \max \{ \tilde{r}_h(r^o, \omega_t), \hat{r}_m(\omega_t) \} & \text{if } r^o > \hat{r}_h(\omega_t) \\
    r^o & \text{otherwise.}
    \end{cases}
\]  

(A.8)

**Condition 1. Ex Ante gridlock:** \( r^o \in [\hat{r}_l(\omega_0), \hat{r}_h(\omega_0)] \). Substituting the expression for \( i \)'s ideal point and rearranging, this implies

\[
    \omega_0 \leq A \alpha_l,
\]

(A.9)

where \( A \equiv (1 + r^o)^2 \).

**Condition 2. Ex post ungridlocked.** \( r^o < \hat{r}_l(\omega_1) \). Substituting and rearranging yields

\[
    \Delta > A \alpha_l - \omega_0 \equiv \Delta'
\]

(A.10)

There are two cases to consider. We proceed by demonstrating that a welfare enhancing shock is ruled out in both.

(a) \( r^*(\omega_1) = \hat{r}_m(\omega_1) \). Some simple algebra reveals:

\[
    \tilde{r}_i(r^o, \omega_1) = \frac{\omega + \Delta}{\alpha_i(1 + r^o)} - 1.
\]

\( r^*(\omega_1) = \hat{r}_m(\omega_1) \) implies \( \hat{r}_m(\omega_1) < \tilde{r}_l(r^o, \omega_1) \). Substituting yields

\[
    \sqrt{\frac{\omega_0 + \Delta}{\alpha_m}} - 1 < \frac{\omega_0 + \Delta}{\alpha_l (1 + r^o)} - 1,
\]

or, rearranging,

\[
    \Delta > \frac{A \alpha_l^2}{\alpha_m} - \omega_0 \equiv \Delta''.
\]

(A.11)

Comparing the expressions for \( \Delta' \) and \( \Delta'' \), the latter exceeds the former if and only if
\( \alpha_l > \alpha_m \). By assumption, \( \hat{r}_l < \hat{r}_m \), implying \( \alpha_l > \alpha_m \). Therefore, \( \Delta'' \) is binding.

**Condition 3(a).** Substituting for aggregate ex post welfare evaluated at \( \hat{r}^m(\omega_1) \) from (A.7), ex post welfare exceeds ex ante welfare if and only if

\[
-\frac{\omega_0}{1 + r^o} - \alpha_m r^o < \alpha_m - 2\sqrt{\alpha_m(\omega_0 + \Delta)}
\]

or, substituting and rearranging

\[
\Delta < \frac{(A\alpha_m - \omega_0)^2}{4A\alpha_m} \equiv \Delta'.
\]  

(A.12)

The set \( (\Delta'', \Delta') \) is nonempty if and only if \( \Delta'' < \Delta' \). Substituting from (A.11) and (A.12) and rearranging yields

\[
\omega_0 > A(2\alpha_l - \alpha_m).
\]

Condition 2 requires \( \omega_0 < A\alpha_l \). There exists a value of \( \omega_0 \) that satisfies both conditions only if \( A(2\alpha_l - \alpha_m) < A\alpha_l \). This simplifies to \( \alpha_l < \alpha_m \), which is ruled out by assumption.

(b) \( r^*(\omega_1) = \hat{r}_l(r^o, \omega_1) \). From part 1, this implies

\[
\Delta < \frac{A\alpha_l^2}{\alpha_m} - \omega_0 \equiv \Delta''.
\]  

(A.13)

Aggregate ex post welfare evaluated at \( r^*(\omega_1) \) simplifies to

\[
\alpha_m - \alpha_l(1 + r^o) - \frac{\alpha_m(\omega_0 + \Delta)}{\alpha_l(1 + r^o)}.
\]

**Condition 3(b).** Comparing this to ex ante aggregate welfare (evaluated at \( \omega = \omega_0 \) and
\( r = r^0 \), the shock is welfare enhancing if and only if

\[
\Delta < \frac{\alpha_l (\alpha_m - \alpha_l)(A - \frac{\omega_m}{\alpha_l})}{\alpha_m}.
\]

The right side of this inequality is strictly negative because \( \alpha_m < \alpha_l \) by assumption. Therefore, there exists no \( \Delta > 0 \) that enhances welfare.