B Online Appendix to “Political Accountability and Sequential Policymaking”

B.1 Equilibrium

Lemma B.1 Consider the model with a fungible budget. An associated SPNE, $s^*$, is robust to small revision costs if and only if

1. $\rho^*(m_1, m_2, a_1, a_2)(\cdot, \cdot) = m_2(\cdot, \cdot)$ for all $(m_1, m_2, a_1, a_2)$;

2. $a_2^*(m_1, m_2, a_1) \in \arg \max_{a_2 \in [0, \pi - a_1]} m_2(a_1, a_2)B + u(\pi - a_1 - a_2)$, for all $(m_1, m_2, a_1)$;

3. $m_2^*(m_1, a_1)(\cdot, \cdot) \in \arg \max_{m_2 \in R} p(a_1, a_2^*(m_1, m_2, a_1))$, for all $(m_1, a_1)$;

4. $a_1^*(m_1) \in \arg \max_{a_1 \in [0, \pi]} m_2^*(m_1, a_1)(a_1, a_2^*(m_1, m_2^*(m_1, a_1), a_1))B + u(\pi - a_1 - a_2^*(m_1, m_2^*(m_1, a_1), a_1))$, for all $m_1$;

5. $m_1^* \in \arg \max_{m_1 \in R} p(a_1^*(m_1), a_2^*(m_1, m_2^*(a_1^*(m_1), a_1), m_1^*(m_1)))$.

Proof. Necessity: Fix an $s^*$ that is robust to small revision costs.

Consider the retention stage. Fix an $\epsilon$. At this stage, the policymaker’s actions are already taken. The Overseer’s action at this stage has no effect on the choice of policymaker actions. However, if $\rho^*(m_1, m_2, a_1, a_2) \neq m_2$, the Overseer suffers a cost $\epsilon$. Hence, sequential rationality requires $\rho^*(m_1, m_2, a_1, a_2) = m_2$ for all $(m_1, m_2, a_1, a_2)$. Since this is true for all $\epsilon$ and the sequence converges, this establishes point 1.

Next consider the policymaker’s strategy in the second policymaking stage. For any $\epsilon$, the policymaker infers that the Overseer will choose $\rho^*(m_1, m_2, a_1, a_2) = m_2$. Hence, the policymaker chooses $a_2^*$ to maximize his expected utility given the retention rule $m_2$ and the history. That is, for any $\epsilon$, $a_2^*(m_1, m_2, a_1) = a_2^*(m_1, m_2, a_1)$ for any $(m_1, m_2, a_1)$. This establishes point 2.

Next consider the Overseer’s strategy in the second policymaking stage. Define $\hat{m}_2(a_1) = \arg \max_{m_2 \in R} p(a_1, a_2^*(m_1, m_2, a_1))$. For any $\epsilon > 0$, given that the action $a_1$ is already taken, the Overseer chooses $m_2$ to maximize her utility net of any revision costs. Thus, she either chooses $m_2 = m_1$ or she chooses $m_2 = \hat{m}_2(a_1)$. For each history $(m_1, a_1)$ with $m_1 \neq \hat{m}_2(a_1)$, she chooses $m_2 = \hat{m}_2(a_1)$ for $\epsilon$ sufficiently small, in particular, she does so if $\max_{m_2 \in R} p(a_1, a_2^*(m_1, m_2, a_1)) - p(a_1, a_2^*(m_1, m_1, a_1)) \geq \epsilon$. Label with $\pi(m_1, a_1)$, the $\epsilon$ that makes this hold with equality. Recall, from the previous paragraph, that $a_2^*(m_1, m_2, a_1) = a_2^*(m_1, m_2, a_1)$ for all $(m_1, m_2, a_1)$. Hence, $\hat{m}_2(a_1)^* = m_2^*(m_1, m_2, a_1)$ for all $(m_1, m_2, a_1)$. 


Thus, it suffices to show that, given that it converges (which we know by hypothesis), the sequence \( \{m_2^*\} \) converges to \( \hat{m}_2(a_1) \). This is clearly true, since \( m_2^*(m_1, a_1) = \hat{m}_2(a_1) \) for all \( \epsilon < \tau(m_1, a_1) \) for all \((m_1, a_1)\). This establishes point 3.

Next consider the policymaker’s strategy in the first policymaking stage. Fix an \( \epsilon > 0 \). Define \( m_2^*(m_1, a_1) \) as whichever of \( m_1 \) and \( \hat{m}_2(a_1) \) maximizes the Overseer’s expected utility at the second policymaking stage. By backward induction, the policymaker knows that for any \((m_1, a_1)\), the Overseer will choose \( m_2^*(m_1, a_1) \) and the policymaker himself will then best respond with \( a_2^*(m_1, m_2^*(m_1, a_1), a_1) \). Hence, the policymaker chooses

\[
a_1^*(m_1) \in \arg\max_{a_1 \in [0, \mathcal{A}]} \mathbb{E}_0[m_2^*(m_1, a_1)(a_1, a_2^*(a_1, m_1, m_2^*(m_1, a_1)))B + u(\mathcal{A} - a_1 - a_2^*(m_1, m_2^*(m_1, a_1)), a_1)].
\]

We need to show that, given that it converges, the sequence \( \{a_1^*(m_1)\} \) converges to \( a_1^*(m_1) \).

Given an \( \epsilon \), for any \((m_1, a_1)\), the Overseer chooses \( m_2^*(m_1, a_1) \) either equal to \( m_1 \) or to \( m_2^*(m_1, a_1) \). Suppose the policymaker anticipates that for his choice of \( a_1^*(m_1) \), we have \( m_2^*(m_1, a_1^*(m_1)) = m_2^*(m_1, a_1^*(m_1)) \). Then \( a_1^*(m_1) \) must be a best response to \( m_2^*(m_1, a_1^*(m_1)) \), which implies \( a_1^*(m_1) = a_1^*(m_1) \). Now suppose the policymaker anticipates that for his choice of \( a_1^*(m_1) \), we have \( m_2^*(m_1, a_1^*(m_1)) = m_1 \neq m_2^*(m_1, a_1^*(m_1)) \). By the fact that \( m_2^* \neq m_2^* \), there exists a second period retention rule that would yield a higher second period action. So for \( \epsilon \) low enough (in particular, \( \epsilon' < \tau(m_1, a_1^*(m_1)) \)) we have that

\[
m_2^*(m_1, a_1^*(m_1)) = m_2^*(m_1, a_1^*(m_1)).
\]

Now, since \( a_1^*(m_1) = a_1^*(m_1) = a_1^*(m_1) \), this takes us back to the first case and establishes point 4.

Finally, consider the Overseer’s strategy in the first policymaking stage. For any \((m_1, a_1)\), \( m_2^*(m_1, a_1) = m_2^*(m_1, a_1) \) for \( \epsilon < \tau(m_1, a_1) \). Moreover, from point 3, \( m_2^*(m_1, a_1) \) is constant in \( m_1 \). Hence, for \( \epsilon \) sufficiently small, backward induction implies that \( a_1^*(m_1) \) is unaffected by the choice of \( m_1 \). Consider such an \( \epsilon \). Suppose \( m_1^* \neq m_2^*(m_1^*, a_1^*(m_1^*)) \). Consider a deviation to a new \( m_1 = m_2^*(m_1^*, a_1^*(m_1^*)) \). This makes the Overseer strictly better off. To see this, note that as already argued, \( a_1^* \) is left unchanged. But the Overseer avoids revising the rule and bearing cost \( \epsilon \). Thus, for \( \epsilon \) sufficiently small, sequential rationality implies that \( m_1^* \in \arg\max_{r \in \mathcal{R}} p(a_1^*(r), a_2^*(r, m_2^*(a_1^*(r), r), a_1^*(r))) \). Now note that \( a_1^* = a_1^* \) and \( a_2^* = a_2^* \). This establishes point 5.

**Sufficiency:** Fix an \( s^* \) satisfying 1–5.

From the necessity proof, the policymaker’s strategy in \( s^* \) is identical to his strategy in a SPNE of a game with any \( \epsilon \). Thus, all we need to show is convergence of the Overseer’s strategy.

First, consider whether \( m_2^* \) converges to \( m_2^* \). Fix a \( \delta > 0 \). We must show that there exists an \( \epsilon \) such that \( |m_2^*(m_1, a_1) - m_2^*(m_1, a_1)| < \delta \) for all \((m_1, a_1)\). By point 3, \( m_2^*(m_1, a_1) = \hat{m}_2(a_1) \) for all \((m_1, a_1)\). Thus, it suffices to show there exists an \( \epsilon \) such that \( |m_2^*(m_1, a_1) -
\( \hat{m}_2(a_1) < \delta \). This is true for all \( \epsilon < \tau(m_1, a_1) \), since for such \( \epsilon \), \( m_2^\epsilon(m_1, a_1) = \hat{m}_2(a_1) \).

Finally, consider whether \( m_1^\epsilon \) converges to \( m_1^\ast \). Fix a \( \delta > 0 \). By sequential rationality, for any \( \epsilon, m_1^\epsilon \in \arg \max_{r \in \mathbb{R}} p(a_1^\ast(r), a_2^\ast(r, m_2^\ast(r, a_1^\ast(r))) \). And, as we saw above, for \( \epsilon \) sufficiently small, \( m_2^\epsilon = m_2^\ast \). Hence, for those same \( \epsilon \), \( m_1^\epsilon = m_1^\ast \).

### B.2 Uncertainty Results

**Lemma B.2** When actions are non-transparent and there is uncertainty over the policy success function, the Overseer’s equilibrium payoffs are weakly higher when the budget is fungible than under any task-specific budget caps.

**Proof.** We first show that, under budget caps \((\pi_1, \pi_2)\), the Overseer’s optimal rule remains \( r(s) = 1 \) and \( r(f) = 0 \). To get a contradiction, suppose that the optimal rule is some other \( r \). Further, suppose \((a_1^\ast(r, \omega), a_2^\ast(r, \omega))\) are the Bureaucrat’s best response to \( r \) at \( \omega \). There are now two cases to consider:

1. Suppose \( a_1^\ast < \pi_1 \) and \( a_2^\ast < \pi_2 \). Then they are characterized by the following first order conditions:

   \[
   \hat{p}_1(a_1^\ast, a_2^\ast, \omega)[r(s) - r(f)] = u'(\overline{A} - a_1^\ast - a_2^\ast)
   \]

   and

   \[
   \hat{p}_2(a_1^\ast, a_2^\ast, \omega)[r(s) - r(f)] = u'(\overline{A} - a_1^\ast - a_2^\ast).
   \]

   It is straightforward that \( a_1^\ast \) and \( a_2^\ast \) are increasing in \( r(s) - r(f) \), so \( r \) was not the optimal rule.

2. Suppose one of \( a_1^\ast \) or \( a_2^\ast \) is equal to the corresponding budget cap. Without loss of generality, let it be \( a_1^\ast = \pi_1 \). Then \( a_2^\ast \) is characterized by the following first order condition:

   \[
   \hat{p}_2(\pi_1, a_2^\ast, \omega)[r(s) - r(f)] = u'(\overline{A} - \pi_1 - a_2^\ast).
   \]

   Clearly \( a_1^\ast \) is non-decreasing and \( a_2^\ast \) is increasing in \( r(s) - r(f) \), so \( r \) was not the optimal rule.

The two points above show that for any \( \omega \), the optimal rule is \( r(s) = 1 \) and \( r(f) = 0 \). Hence, this is the optimal rule, ex ante, for the Overseer.

Since, by an argument identical to the proof of Proposition 5.1, the allocation across tasks is efficient under a fungible budget, it suffices to show that total effort is weakly higher under a fungible budget than under budget caps.
Label the Bureaucrat’s equilibrium total spending, given the retention rule \((r(s) = 1, r(f) = 0)\), under a fungible budget \(A^F\). Label the Bureaucrat’s equilibrium total spending, given the retention rule \((r(s) = 1, r(f) = 0)\), under a particular budget cap \(A^C(\pi_1, \bar{A} - \pi_2)\).

Now, define \(\hat{A}_1(\omega, \pi_1)\) as the lowest amount of total spending such that, at \(\omega\), the efficient allocation involves \(a_1^e(A) \geq \pi_1\). That is,

\[
a_1^e(\hat{A}_1(\omega, \pi_1)) = \pi_1.
\]

Since \(a_1^e(A)\) is strictly increasing in \(A\), it is straightforward that \(\hat{A}_1(\omega, \pi_1)\) exists and is unique for each \((\omega, \pi_1)\). Define \(\hat{A}_2(\omega, \pi_2)\) analogously.

There are two cases to consider.

1. Suppose that \(A^C \leq \min\{\hat{A}_1(\omega, \pi_1), \hat{A}_2(\omega, \pi_2)\}\). Then the efficient division of the Bureaucrat’s total spending is unconstrained by the budget caps. As such, by an argument identical to the proof of Proposition 5.1, the Bureaucrat will choose this efficient division. Hence, the Bureaucrat faces an identical problem to the situation with a fungible budget and total spending is the same under each, as required.

2. Suppose \(A^C > \min\{\hat{A}_1(\omega, \pi_1), \hat{A}_2(\omega, \pi_2)\}\). Without loss of generality, assume \(\hat{A}_1(\omega, \pi_1) = \min\{\hat{A}_1(\omega, \pi_1), \hat{A}_2(\omega, \pi_2)\}\). The best responses under this budget cap, then, are \((\pi_1, A^C - \pi_1)\). There are now two sub-cases to consider:

   (a) \(A^F < \min\{\hat{A}_1(\omega, \pi_1), \hat{A}_2(\omega, \pi_2)\}\). This implies that \(a_1^e(A^F) < \pi_1\) and \(a_2^e(A^F) < \pi_2\). But this implies that the allocation chosen under the fungible budget was feasible under budget caps. Since it was optimal in the unconstrained problem, and it satisfies the constraints, it remains optimal in the constrained problem. This contradicts that \((\pi_1, A^C - \pi_1)\) was a best response. Hence, it is not possible for \(A^C > \min\{\hat{A}_1(\omega, \pi_1), \hat{A}_2(\omega, \pi_2)\}\) and \(A^F < \min\{\hat{A}_1(\omega, \pi_1), \hat{A}_2(\omega, \pi_2)\}\).

   (b) Now consider \(A^F \geq \min\{\hat{A}_1(\omega, \pi_1), \hat{A}_2(\omega, \pi_2)\}\). This implies that \(a_1^e(A^F) \geq \pi_1\). Since second allocations are interior, they satisfy first order conditions. In the case of a fungible budget we have:

\[
\hat{p}_2(a_1^e(A^F), a_2^F, \omega)B = u'(\bar{A} - A^F). \tag{8}
\]

In the case of the budget caps we have:

\[
\hat{p}_2(\pi_1, a_2^C, \omega)B = u'(\bar{A} - A^C). \tag{9}
\]
Since we already know that \( a_1^*(A^F) \geq \pi_1 \), to show that \( A^F \geq A^C \) it suffices to show that \( a_2^F \geq a_2^C \).

To get a contradiction, assume \( a_2^F < a_2^C \). From the concavity of \( \hat{p} \) we have:

\[
\hat{p}_2(\overline{a}_1, a_2^C, \omega) < \hat{p}_2(\overline{a}_1, a_2^F, \omega).
\]

This fact, combined with the weak complementarity of \( a_1 \) and \( a_2 \) in \( \hat{p} \), implies

\[
\hat{p}_2(\overline{a}_1, a_2^C, \omega) < \hat{p}_2(a_1^F, a_2^F, \omega).
\]

This inequality, implies that the left-hand side of the first-order condition in Equations 8 is larger than the left-hand side of the first-order condition in 9, which implies \( a_2^F > a_2^C \), a contradiction.

The following generalizes Figure 2.

**Proposition B.1** Let

\[
\hat{p}(a_1, a_2, \omega) = \omega f(a_1) + (1 - \omega) f(a_2),
\]

with \( f(a) = a^{1/k} \) and \( k > 1 \). Further, assume \( \overline{A} = 1 \) and \( u(x) = \sqrt{x} \). Then there exists a \( \overline{k} > 1 \) such that for any

\[
k \in (1, \overline{k}),
\]

there exists a \( \overline{B}(k) < 1 \) such that if

\[
B \in (\overline{B}(k), 1],
\]

then the Overseer prefers non-transparency to transparency.

**Proof.** From Proposition A.1, it suffices to show that there exists an \( (\overline{k}, \overline{B}(k)) \) such that, for \( k \in (1, \overline{k}) \) and \( B \in (\overline{B}(k), 1) \), we have

\[
\hat{A}^{NT}(1) > \frac{A^{\text{max}}}{2}.
\]
Note, first, that $A^{\text{max}}$ satisfies

$$B + \sqrt{1 - A^{\text{max}}} = 1 \Rightarrow A^{\text{max}} = 1 - (1 - B)^2. \quad (10)$$

Let $\hat{A}^{NT}(1)$ be the limit of $\hat{A}^{NT}$ as $\varpi \to 1$. Then, $\hat{A}^{NT}(1)$ satisfies the following first-order condition:

$$\frac{B}{k} (\hat{A}^{NT}(1))^{1-k} = \frac{1}{2 \sqrt{\hat{A}^{NT}(1)}}.$$

Rearranging, we have that $\hat{A}^{NT}(1)$ satisfies

$$\left( \hat{A}^{NT}(1) \right)^{\frac{2(k-1)}{k}} - \left( \hat{A}^{NT}(1) \right)^{\frac{2(1-k)+k}{k}} = \frac{k^2}{4B^2}. \quad (11)$$

Looking at Equation 11, the limit of the left-hand side as $k$ goes to 1 is $1 - \hat{A}^{NT}(1)$. The limit of the right-hand side as $k$ goes to 1 is $\frac{1}{4B^2}$. Hence,

$$\lim_{k \to 1} \lim_{\varpi \to 1} \hat{A}^{NT} = 1 - \frac{1}{4B^2}.$$

It is straightforward from Equation 10 that $A^{\text{max}}$ is not a function of $k$.

Since actions and payoffs are continuous in $k$, the above shows that, for any $B$, if $k$ is sufficiently close to 1, $\hat{A}^{NT}(1) > A^{\text{max}}/2$ as long as

$$1 - \frac{1}{4B^2} > \frac{1 - (1 - B)^2}{2}.$$

Both sides of this inequality are continuous in $B$. Hence, it now suffices to show that the limit as $B$ goes to 1 of the left-hand side is larger than the right-hand side. As $B$ goes to 1, the left-hand side of this inequality goes to $\frac{3}{4}$, while the right-hand side goes to $\frac{1}{2}$. ■

**B.3 Budget Caps**

The following result provides necessary and sufficient conditions for a strategy profile to be a SPNE that is robust to small revision costs when there are budget caps.

**Lemma B.3** An SPNE, $s^*$, is robust to small revision costs if and only if

1. $\rho^*(m_1, m_2, a_1, a_2)(\cdot, \cdot, \cdot) = m_2(\cdot, \cdot, \cdot)$ for all $(m_1, m_2, a_1, a_2)$;

2. $a_2^*(m_1, m_2, a_1) \in \arg\max_{a_2 \in [0, \overline{a}]} \mathbb{E}_O[m_2(a_1, a_2, O)B + u(\overline{A} - a_1 - a_2)]$, for all $(m_1, m_2, a_1)$;
We need to show that, given that it converges, the sequence \(a\) responds with \(a_{\text{max}}\) establishes point 2.

Proof. Necessity: Fix an \(s^*\) that is robust to small revision costs.

Consider the retention stage. Fix an \(\epsilon\). At this stage, the Bureaucrat’s actions are already taken. The Overseer’s action at this stage has no effect on the choice of Bureaucrat actions. However, if \(\rho'(m_1, m_2, a_1, a_2) \neq m_2\), the Overseer suffers a cost \(\epsilon\). Hence, sequential rationality requires \(\rho'(m_1, m_2, a_1, a_2) = m_2\) for all \((m_1, m_2, a_1, a_2)\). Since this is true for all \(\epsilon\) and the sequence converges, this establishes point 1.

Next consider the Bureaucrat’s strategy in the second policymaking stage. For any \(\epsilon\), the Bureaucrat infers that the Overseer will choose \(\rho'(m_1, m_2, a_1, a_2) = m_2\). Hence, the Bureaucrat chooses \(a_{\text{max}}\) to maximize his expected utility given the retention rule \(m_2\) and the history. That is, for any \(\epsilon\), \(a_2^s(m_1, m_2, a_1) = a^*_2(m_1, m_2, a_1)\) for any \((m_1, m_2, a_1)\). This establishes point 2.

Next consider the Overseer’s strategy in the second policymaking stage. Define \(\tilde{m}_2(a_1) = \arg\max_{m_2} p(a_1, a_{\text{max}}^s(m_1, m_2, a_1))\). For any \(\epsilon > 0\), given that the action \(a_1\) is already taken, the Overseer chooses \(m_2\) to maximize her utility net of any revision costs. Thus, she either chooses \(m_2 = m_1\) or she chooses \(m_2 = \tilde{m}_2(a_1)\). For each history \((m_1, a_1)\) with \(m_1 \neq \tilde{m}_2(a_1)\), she chooses \(m_2 = \tilde{m}_2(a_1)\) for \(\epsilon\) sufficiently small, in particular, she does so if \(\max_{m_2} p(a_1, a_{\text{max}}^s(m_1, m_2, a_1)) - p(a_1, a_{\text{max}}^s(m_1, m_1, a_1)) \geq \epsilon\). Label with \(\tilde{\pi}(m_1, a_1)\), the \(\epsilon\) that makes this hold with equality. Recall, from the previous paragraph, that \(a_{\text{max}}^s(m_1, m_2, a_1) = a_{\text{max}}^s(m_1, m_2, a_1)\) for all \((m_1, m_2, a_1)\). Hence, \(\tilde{m}_2(a_1) = m_{\text{max}}^s(m_1, m_2, a_1)\) for all \((m_1, m_2, a_1)\). Thus, it suffices to show that, given that it converges (which we know by hypothesis), the sequence \(\{m_{\text{max}}^s\}\) converges to \(\tilde{m}_2(a_1)\). This is clearly true, since \(m_{\text{max}}^s(m_1, a_1) = \tilde{m}_2(a_1)\) for all \(\epsilon < \tilde{\pi}(m_1, a_1)\) for all \((m_1, a_1)\). This establishes point 3.

Next consider the Bureaucrat’s strategy in the first policymaking stage. Fix an \(\epsilon > 0\). Define \(m_{\text{max}}^s(m_1, a_1)\) as whichever of \(m_1\) and \(\tilde{m}_2(a_1)\) maximizes the Overseer’s expected utility at the second policymaking stage. By backward induction, the Bureaucrat knows that for any \((m_1, a_1)\), the Overseer will choose \(m_{\text{max}}^s(m_1, a_1)\) and the Bureaucrat himself will then best respond with \(a_{\text{max}}^s(m_1, m_{\text{max}}^s(m_1, a_1), a_1)\). Hence, the Bureaucrat chooses \(a_{\text{max}}^s(m_1)\) for all \((m_1, a_1)\). We need to show that, given that it converges, the sequence \(\{a_{\text{max}}^s(m_1)\}\) converges to \(a_{\text{max}}^s(m_1)\).
Given an \( \epsilon \), for any \((m_1, a_1)\), the Overseer chooses \( m'_2(m_1, a_1) \) either equal to \( m_1 \) or to \( m_2(m_1, a_1) \). Suppose the Bureaucrat anticipates that for his choice of \( a'_1(m_1) \), we have \( m'_2(m_1, a'_1(m_1)) = m^*_2(m_1, a'_1(m_1)) \). Then \( a'_1(m_1) \) must be a best response to \( m^*_2(m_1, a'_1(m_1)) \), which implies \( a'_1(m_1) = a^*_1(m_1) \). Now suppose the Bureaucrat anticipates that for his choice of \( a'_1(m_1) \), we have \( m'_2(m_1, a'_1(m_1)) = m_1 \neq m^*_2(m_1, a'_1(m_1)) \). By the fact that \( m'_2 \neq m^*_2 \), there exists a second-stage retention rule that would yield a higher second-stage action. So for \( \epsilon \) low enough (in particular, \( \epsilon' < \bar{\epsilon}(m_1, a'_1(m_1)) \)) we have that \( m'_2(m_1, a'_1(m_1)) = m^*_2(m_1, a'_1(m_1)) \). Now, since \( a'_1(m_1) = a^*_1(m_1) \), this takes us back to the first case and establishes point 4.

Finally, consider the Overseer’s strategy in the first policymaking stage. For any \((m_1, a_1), m'_2(m_1, a_1) = m^*_2(m_1, a_1) \) for \( \epsilon \leq \bar{\epsilon}(m_1, a_1) \). Moreover, from point 3, \( m^*_2(m_1, a_1) \) is constant in \( m_1 \). Hence, for \( \epsilon \) sufficiently small, backward induction implies that \( a'_1(m_1) \) is unaffected by the choice of \( m_1 \). Consider such an \( \epsilon \). Suppose \( m'_1 \neq m^*_1(m'_1, a'_1(m'_1)) \). Consider a deviation to a new \( m_1 = m'_2(m'_1, a'_1(m'_1)) \). This makes the Overseer strictly better off. To see this, note that as already argued, \( a'_1 \) is left unchanged. But the Overseer avoids revising the rule and bearing cost \( \epsilon \). Thus, for \( \epsilon \) sufficiently small, sequential rationality implies that \( m'_1 \in \arg\max_{r \in R} p(a'_1(r), a'_2(r, m'_2(a'_1(r), r), a'_1(r))) \). Now note that \( a'_1 = a^*_1 \) and \( a'_2 = a^*_2 \). This establishes point 5.

**Sufficiency:** Fix an \( s^* \) satisfying 1–5.

From the necessity proof, the Bureaucrat’s strategy in \( s^* \) is identical to his strategy in a SPNE of a game with any \( \epsilon \). Thus, all we need to show is convergence of the Overseer’s strategy.

First, consider whether \( m'_2 \) converges to \( m^*_2 \). Fix a \( \delta > 0 \). We must show that there exists an \( \epsilon \) such that \( |m'_2(m_1, a_1) - m^*_2(m_1, a_1)| < \delta \), for all \((m_1, a_1)\). By point 3, \( m^*_2(m_1, a_1) = \hat{m}_2(a_1) \) for all \((m_1, a_1)\). Thus, it suffices to show there exists an \( \epsilon \) such that \( |m'_2(m_1, a_1) - \hat{m}_2(a_1)| < \delta \). This is true for all \( \epsilon < \bar{\epsilon}(m_1, a_1) \), since for such \( \epsilon \), \( m'_2(m_1, a_1) = \hat{m}_2(a_1) \).

Finally, consider whether \( m'_1 \) converges to \( m^*_1 \). Fix a \( \delta > 0 \). By sequential rationality, for any \( \epsilon, m'_1 \in \arg\max_{r \in R} p(a^*_1(r), a^*_2(r, m^*_2(a^*_1(r), r), a^*_1(r))) \). And, as we saw above, for \( \epsilon \) sufficiently small, \( m'_2 = m^*_2 \). Hence, for those same \( \epsilon \), \( m'_1 = m^*_1 \).

**B.3.1 Proof of Proposition 5.3**

**Lemma B.4** There exists \( m_2 \in R \) such that the Bureaucrat’s best response is to allocate \( a'_2 \leq \bar{a}_2 \), following a history \((m_1, m_2, a_1)\), if and only if \( a'_2 \leq \hat{a}_2(a_1) \) implicitly defined by

\[
B + u(\bar{A} - a_1 - \hat{a}_2) = u(\bar{A} - a_1).
\]

(12)
Proof.

From Lemma B.3, the second allocation can be $a^*_2$ following the history $(m_1,m_2,a_1)$ if
\[
\mathbb{E}_O[m_2(m_1,a_1)(a_1,a^*_2,O)]B + u(\overline{A} - a_1 - a^*_2) \geq \mathbb{E}_O[m_2(m_1,a_1)(a_1,a''_2,O)]B + u(\overline{A} - a_1 - a''_2),
\]
for all $a''_2 \in [0,\overline{a}_2]$. It is easiest to induce the Bureaucrat to choose $a^*_2$ by setting the retention probability to 0 for all other choices. Suppose $m_2$ assigns probability $r$ to retention following $a'_2$ and 0 for all other choices. Then $a^*_2$ is a best response if and only if $rB + u(\overline{A} - a_1 - a^*_2) \geq u(\overline{A} - a_1 - a'_2)$, for all $a'_2 \in [0,\overline{a}_2]$. Clearly, now, the binding constraint for the Bureaucrat is $a''_2 = 0$. Hence, $a^*_2$ is a best response if and only if $rB + u(\overline{A} - a_1 - a^*_2) \geq u(\overline{A} - a_1)$. Setting $r = 1$ now establishes the result. □

Lemma B.5 Let $\hat{a}_1$ be the maximal $a_1$ such that it is feasible to extract the full second task budget from the Bureaucrat:

\[
\frac{u(\overline{A} - \hat{a}_1) - u(\overline{A} - \hat{a}_1 - \overline{a}_2)}{B} = 1.
\] (13)

In any equilibrium, at any history $(m_1,a_1)$:

- If $a_1 < \hat{a}_1$, then the Overseer announces a rule, $m_2(m_1,a_1,a_2)$ such that
  \[
  \max_{a_2} \left[ m_2(m_1,a_1,a_2)(a_1,a_2,O)B + u(\overline{A} - a_1 - a_2) \right] = \overline{a}_2
  \]
- If $a_1 \geq \hat{a}_1$, then the Overseer announces a rule, $m_2(m_1,a_1,a_2)$ that certainly retains if $a_2 = \hat{a}_2(a_1)$ and certainly replaces if $a_2 < \hat{a}_2(a_1)$.

Proof. Follows from Lemmas B.3 and B.4. □

Lemma B.6 If $\hat{a}_1 > 0$, then there does not exist an equilibrium in which the Bureaucrat chooses $a_1 \geq \hat{a}_1$.

Proof. By Lemma B.5, if $a_1 \geq \hat{a}_1$, then the Bureaucrat will be induced, in the second policymaking stage, to choose $\hat{a}_2(a_1)$ and will be retained for certain. Hence, for any $a_1 \geq \hat{a}_1$, the Bureaucrat’s payoff is $B + u(\overline{A} - a_1 - \hat{a}_2(a_1))$. Equation 12 implies that the Bureaucrat’s payoff can be written as $u(\overline{A} - a_1)$. This is strictly decreasing in $a_1$. Hence, the payoff from choosing $a_1 \geq \hat{a}_1$, when the Overseer uses a retention rule that is consistent with a SPNE that is robust to small revisions costs, is bounded above by $u(\overline{A} - \hat{a}_1)$. But the Bureaucrat can deviate to $(0,0)$ and make a payoff that is bounded below by $u(\overline{A}) > u(\overline{A} - \hat{a}_1)$. Thus, $a_1 \geq \hat{a}_1$ is never part of a best response. □
Lemma B.7 For any $a_1 < \hat{a}_1$, all of the following are true:

1. For any retention rule that is consistent with Lemma B.5, the Bureaucrat’s payoff from choosing $(a_1, \overline{a}_2)$ is bounded below by $u(\overline{A} - a_1)$ and bounded above by $B + u(\overline{A} - a_1 - \overline{a}_2)$.

2. There exists a retention rule that induces the Bureaucrat to choose $a_2 = \overline{a}_2$ following $a_1$ while providing the Bureaucrat with a payoff of $u(\overline{A} - a_1)$.

3. There exists a retention rule that induces the Bureaucrat to choose $a_2 = \overline{a}_2$ following $a_1$ while providing the Bureaucrat with a payoff of $B + u(\overline{A} - a_1 - \overline{a}_2)$.

Proof. (i) From Lemma B.5, following $a_1 < \hat{a}_1$, any retention rule that is consistent with a SPNE that is robust to small revision costs must induce $a_2 = \overline{a}_2$. To find a lower bound on the associated payoff, consider the lowest retention probability that the Overseer can offer while still inducing $\overline{A}$. Clearly, the Overseer should offer certain non-retention for any $a_2 < \overline{a}_2$. Let $q(a_1, \overline{a}_2)$ be the retention probability at $(a_1, \overline{a}_2)$. The Bureaucrat’s best response will be to choose $a_2 = \overline{a}_2$, as long as this retention probability satisfies $q(a_1, \overline{a}_2)B + u(\overline{A} - a_1 - \overline{a}_2) \geq u(\overline{A} - a_1)$. Call the smallest retention probability that satisfies this condition $\hat{q}(a_1, \overline{a}_2)$. It satisfies

$$\hat{q}(a_1, \overline{a}_2)B + u(\overline{A} - a_1 - \overline{a}_2) = u(\overline{A} - a_1).$$

Thus, the lower bound on payoffs from the pair $(a_1, \overline{a}_2)$ is $u(\overline{A} - a_1)$, as required.

The highest payoff possible from a pair $(a_1, \overline{a}_2)$ is $u(\overline{A} - a_1)$, as required.

(ii) The argument above shows that a rule that assigns retention probability $\hat{q}(a_1, \overline{a}_2)$ to the pair $(a_1, \overline{a}_2)$ and retention probability 0 to any pair $(a_1, a_2)$ with $a_2 \neq \overline{a}_2$ does so.

(iii) Consider the rule that assigns retention probability 1 to the pair $(a_1, \overline{a}_2)$ and retention probability 0 to any pair $(a_1, a_2)$ with $a_2 \neq \overline{a}_2$. Since $1 > \hat{q}(a_1, \overline{a}_2)$, this rule induces $a_2 = \overline{a}_2$ following $a_1$. Moreover, the argument from point 1 establishes that it induces the payoff required.

Lemma B.8 The highest $a_1$ that the Overseer can induce as a best response to a retention rule that is consistent with Lemma B.5 is $\tilde{a}_1$ given by:

$$B + u(\overline{A} - a_1 - \overline{a}_2) = u(\overline{A}).$$

Proof. The Overseer’s problem is to choose the retention rule that maximizes $a_1$, subject to the constraint that for all $a_1 < \hat{a}_1$, the rule induces $a_2 = \overline{a}_2$. Suppose the Overseer wants to
send a message \( m_1 \) that she will not have an incentive to revise at any history and that will induce the Bureaucrat to choose a pair \((a'_1, \bar{\sigma}_2)\) for some \( a'_1 < \hat{a}_1 \). The strongest incentive she can credibly give to the Bureaucrat is to make all other pairs \((a_1, \bar{\sigma}_2)\) as unattractive as possible (subject to \( a_2 = \bar{\sigma}_2 \) being a best response following each \( a_1 < \hat{a}_1 \)) while making \((a'_1, \bar{\sigma}_2)\) as attractive as possible.

From Lemma B.7, the least attractive the Overseer can make a pair \((a_1, \bar{\sigma}_2)\) without violating the constraint leaves the Bureaucrat with a payoff of \( u(A - a_1) \). Suppose the Overseer does this. Given this, for the Bureaucrat, the most attractive \((a'_1, \bar{\sigma}_2)\) without violating the constraint leaves the Bureaucrat with a payoff of \( B + u(A - a'_1 - \bar{\sigma}_2) \).

Taken together, these arguments imply that the Overseer can credibly announce a rule that is consistent with Lemma B.5 and induces the Bureaucrat to choose \((a'_1, \bar{\sigma}_2)\) if and only if:

\[
B + u(A - a'_1 - \bar{\sigma}_2) \geq u(A),
\]

as required.

**Proof of Proposition 5.3.** Suppose \( B \leq u(A) - u(\bar{\bar{\sigma}} - \bar{\sigma}) \). This implies \( \bar{\sigma}_2 \geq A^{\text{max}} \). Further, it implies \( \hat{a}_1 < 0 \). Hence, for any \( a_1 \), by Lemma B.6, the Overseer will announce a rule that induces \( a_2 = \hat{a}_2(a_1) \). Note from the definition of \( \hat{a}_2(a_1) \) that, in this case, \( \hat{a}_2(a_1) \leq \bar{\sigma}_2 \). Hence, in order to induce \( \hat{a}_2(a_1) \), the Overseer must reward the choice of \( \hat{a}_2(a_1) \) with certain retention. Given this, clearly the Bureaucrat’s best response in the first policymaking stage is to choose \( a_1 = 0 \). Now the result follows from the definition in Equation 12, which implies that \( \hat{a}_2(0) = A^{\text{max}} \).

Suppose \( B \in (u(\bar{\bar{A}}) - u(\bar{\bar{\sigma}}_2), u(\bar{\bar{A}}) - u(0)) \). This implies \( \bar{\sigma}_2 < A^{\text{max}} \). In this case, \( \hat{a}_1 > 0 \). Lemmas B.5 and B.7 thus show that the Overseer will announce and use a retention rule that induces \( a_2 = \bar{\sigma}_2 \). Lemma B.8 shows that the Overseer will announce and use a retention rule that induces \( a_1 = \bar{\sigma} \). Comparing Equations 1 and 14 implies that \( \bar{\sigma}_2 = A^{\text{max}} - \bar{\sigma}_2 \).