The study of money and general equilibrium deals with the integration of monetary theory and the classical theory of value. It includes such topics as the role of money in exchange, the determination of the price level, and the ‘real’ effects of money on the allocation of goods and services.

The general equilibrium theory of value, as developed by Walras (1874–77) and his followers, determines the relative prices of goods in terms of non-monetary factors such as technology, preferences, and endowments. Monetary factors are used to determine the nominal price level once relative prices have been determined. Relative prices are determined by the market-clearing conditions for goods whereas the general price level is determined by the market-clearing condition for money. Given a vector of nominal prices \( p = (p_1, \ldots, p_T) \), the market excess demand functions can be denoted by \( f(p) = (f_1(p), \ldots, f_H(p)) \), where \( p_h \) denotes the nominal price of good \( h \) and \( f_h(p) \) denotes the market excess demand for good \( h \). The functions \( f(p) \) are assumed to be homogeneous of degree zero in nominal prices:

\[
f(p) = f(tp),
\]

for any positive scalar \( t > 0 \). The market-clearing conditions for goods require that the excess demand for each good vanishes at the equilibrium price vector \( p^* \), that is \( f(p^*) = 0 \). These conditions can at most determine relative prices, because if \( p^* \) is an equilibrium price vector, then so is \( tp^* \), for any positive scalar \( t > 0 \).

To determine the nominal price level, a demand function for money is introduced. The aggregate demand for money is assumed to be a function of prices \( M(p) \). Money demand is homogeneous of degree one in prices:

\[
M(tp) = tM(p),
\]

for any price vector \( p \) and any scalar \( t > 0 \). For any vector of nominal prices \( p^* \) satisfying the goods market-clearing condition \( f(p^*) = 0 \), there is a unique value of \( t > 0 \) such that

\[
M(tp^*) = \bar{M},
\]

where \( \bar{M} > 0 \) is the exogenous money supply. Thus, once relative prices have been determined by the real factors, the level of nominal prices is determined by monetary factors. This doctrine, which became known as the classical dichotomy, characterized the classical (pre-Keynesian) thinking about monetary economics (see Fisher, 1963, for example).

The integration of monetary theory and the theory of value was stimulated by the appearance of Keynes’s General Theory (Keynes, 1936). Pigou (1943) argued that the demand for goods could not be homogeneous of degree zero in prices, because a general fall in prices would increase the real value of money and the wealth effect would in turn increase demand for goods. The Pigou effect (the effect of a general fall in prices on the aggregate demand for goods) is a special case of the real balance effect: that is, the effect of any change in real balances on the aggregate demand for goods. In an attempt to make sense of Keynes’s short period analysis, Hicks (1946) introduced the concept of temporary equilibrium, in which prices adjust to clear markets in a particular time period, taking as given expectations about prices in future periods. Building on the work of Hicks and Pigou, Patinkin (1965) argued that the real balance effect is essential for the existence and stability of equilibrium. The classical writers assumed that the market excess demand func-
tions satisfy Say’s Law, that is, the value of excess demands for goods sum to zero or

\[ p \cdot f(p) = 0, \]

for any price vector \( p \). However, Patinkin pointed out that Walras’s Law should also be satisfied: that is, the value of the excess demands for goods and money should sum to zero, or

\[ p \cdot f(p) + M(p) - \bar{M} = 0, \]

for any price vector \( p \). Say’s Law and Walras’s Law together imply that

\[ M(p) = \bar{M}, \]

for any price vector \( p \). Then homogeneity of the excess demand function \( f(p) \) once again implies that, if \( p^* \) is a market-clearing price vector, so is \( t p^* \) for any \( t > 0 \) and the price level is once again undetermined. To avoid this indeterminacy, Patinkin argued that there must be a real balance effect: a change in the general price level implies a change in real balances, and hence a change in wealth which must change the demand for commodities. Thus, in a monetary economy the excess demand for goods \( f(p, M) \) is a (homogeneous of degree zero) function of nominal prices and the money supply.

Hahn (1965) pointed out another problem in the theory of monetary equilibrium, viewed from the Walrasian perspective. The problem was the lack of a proof that money has positive value in equilibrium. Hahn observed that the uses of money that might be expected to give rise to a positive demand for money all require money to have positive value in exchange. If the value of money were zero, the economy would be identical to a barter economy. Under the usual assumptions on the excess demand functions, such a non-monetary economy would possess an equilibrium, but it would not be a monetary equilibrium, because money would have no role in exchange.

Grandmont (1983) provided an elegant solution to the problem posed by Hahn (1965). He showed that, while the real balance effect might be necessary, it was not sufficient for the existence of an equilibrium in which the value of money is positive. A strong intertemporal substitution effect is needed as well. Consider an economy in which there are two periods (the present and the future). In the first period, agents buy and sell goods for immediate consumption. They also demand money as a store of value, which they hold until the following period. The value of money is given by an indirect utility function \( v(m, p') \), where \( m > 0 \) is the amount of money held until the future and \( p' \) is the vector of future nominal prices. An agent’s expectations are represented by a probability measure \( \mu \) on the space of price vectors. Expectations of future prices depend on current prices \( p \) via the expectation function \( \mu = \psi(p) \). Then the expected utility associated with the cash balance \( m \) is simply the expected value of \( v(m, p') \), conditional on the current price vector \( p \):

\[ v(m, p) = \int v(m, p') d\psi(p). \]

Let \( u(x) \) denote the utility associated with the consumption of a vector of current goods \( x \). Then the agent seeks to maximize

\[ u(x) + v(m, p) \]

subject to the budget constraint
where \( e \) is the agent’s endowment of goods and \( \tilde{m} \) his endowment of money. The crucial assumption (sufficient condition) for the existence of an equilibrium in which money has a positive value is that the expectation function \( \psi(p) \) satisfies the uniform tightness property: for any number \( \epsilon > 0 \) and for every current price vector \( p \), there is a compact set \( K \) in the space of positive prices such that \( \psi(p) \) assigns probability at least \( 1 - \epsilon \) to the event that the future price vector \( p' \) belongs to \( K \).

While the classical dichotomy cannot hold in the short run, Archibald and Lipsey (1958) argued that it would hold in the long run because the allocation of money balances is endogenous in the long run. This gave rise to the study of stationary states (see Grandmont, 1983).

**The cash-in-advance constraint**

Introduced by Clower (1967), the cash-in-advance constraint provides a simple motivation for the use of money as a medium of exchange. Lucas (1980) derives the cash-in-advance constraint as follows. Every household is assumed to consist of two agents, one of whom is responsible for selling the household’s endowment of goods (for example, supplying labour) and the other is responsible for purchasing goods. At the beginning of each day, the seller sets off for the market with a bundle of goods to sell, while the buyer sets of for a different set of markets to buy the goods they need. Following Clower’s dictum that ‘money buys goods and goods buy money but goods do not buy goods’, the buyer needs to have a stock of money at the beginning of the day. The money earned by the seller is not available until the end of the day, so the buyer’s purchases are constrained by the amount of money she has at the beginning of the day. The money brought home by the seller must be held until the next day. If \( \tilde{m} \) is the amount of money held initially and \( m \) is the amount carried forward to the next day, the budget constraint can be written as

\[
p \cdot x + m \leq p \cdot e + \tilde{m},
\]

and the cash-in-advance constraint can be written as

\[
p \cdot (x - e)^+ \leq \tilde{m},
\]

where \( x^+ \) denotes the vector consisting of the non-negative part of the vector \( x \).

Grandmont and Younes (1973) used a cash-in-advance constraint to study the efficiency of monetary equilibrium. They considered stationary equilibria of an infinite-horizon, pure-exchange economy in which a finite number of individuals \( i = 1, \ldots, I \) maximize the discounted sum of utilities

\[
\sum_{t=0}^{\infty} \delta^{t-t_0} u_i(x_i(t))
\]

subject to a sequence of budget constraints and a cash-in-advance constraint in the form

\[
p(t) \cdot (x(t) - e)^+ + kp \cdot (x(t) - e)^- \leq m(t - 1),
\]

where \( 0 \leq k \leq 1 \). For \( k = 0 \) this constraint reduces to the Clower–Lucas version. Grandmont and Younes established Friedman’s optimum quantity of money result: any laissez-faire, stationary equilibrium of this economy is Pareto inefficient but, if the rate of price deflation equals the subjective rate as time preference, this is sufficient to guarantee that equilibrium is efficient. Grandmont and Laroque (1975) also showed that the payment of interest on money has no effect on efficiency. More precisely, it is the gap between the
inflation rate and the interest rate which has an effect, and this is attributable to the lump-sum taxes rather than the interest payments.

The cash-in-advance constraint has played an important role in macroeconomics, particularly in the study of the effect of fiscal and monetary policy (see, for example, Lucas and Stokey, 1983; 1987; Sargent, 1987).

Financial securities

The classical model of general competitive equilibrium assumes that markets are complete. Hart (1975) showed that, with incomplete markets, the existence of equilibrium is no longer guaranteed and the fundamental theorems of welfare economics no longer hold. In Hart’s model, incomplete markets are represented by trade in real securities, which are promises to deliver bundles of commodities at some future date and event. Cass (2006) and Werner (1985) introduced financial securities, whose payoffs are denominated in units of money, and showed that this resolved the existence problem. However, as Balasko and Cass (1989) and Geanakoplos and Mas-Colell (1989) showed, financial securities also introduced indeterminacy of equilibrium. The problem is that a change in the price level in some state changes the real purchasing power of money and hence changes the real payoffs of the financial securities. Magill and Quinzii (1992) pointed out that the indeterminacy arises from the fact that ‘money’ serves only as a unit of account in the Cass–Werner model. Money has no role in exchange or savings and investment, and hence there is no well defined demand for money.

To address this problem, Magill and Quinzii introduce a cash-in-advance constraint in the spirit of Clower (1967). There are two dates, $t=0,1$, and $S$ states of nature, $s=1,...,S$. The state is unknown at date 0; the true state is revealed at date 1. It is convenient to treat the situation at date 0 as another state, denoted $s=0$. Then each period $s$ is divided into three sub-periods, denoted $s_1$, $s_2$, and $s_3$. In sub-period $s_1$, agents sell their entire endowment of money to a central exchange and receive money instead. In sub-period $s_2$, they invest in financial securities (at date 0) and receive dividends (at date 1). In sub-period $s_3$, they use money to purchase goods from the central exchange. The separation of the sale and purchase of goods between sub-periods $s_1$ and $s_3$ forces agents to hold money in equilibrium. Money can also be used to store wealth between periods 0 and 1, but agents will do this only if they anticipate deflation. The supply of money is determined exogenously by the government.

Three main results were established by Magill and Quinzii. First, they showed that, generically in endowments and money supply, an economy has a finite number of locally unique monetary equilibria. This means that equilibrium is locally determinate: the well-defined demand for money has eliminated the indeterminacy of the price level. Second, if money is used as a medium of exchange only, local changes in the money supply have no real effects if the asset markets are complete – changes in the money supply will change the price level but this will have no effect on the real allocation as long as markets are complete – whereas, if markets are incomplete, local changes in money supply translate into an $S-1$ dimensional submanifold of real allocations. When markets are incomplete, any change in the price level implies a change in the real payoffs of the securities, and this translates into a real change in the allocation. Finally, if money is used as a store of value, local changes in the money supply translate into an $S$-dimensional submanifold of real allocations in the case of both complete and incomplete markets. This follows because the use of money as a store of value to transfer wealth
between periods implies that the real allocation is directly impacted by changes in the real payoffs from holding money.

A related study by Geanakoplos and Dubey (1992) addresses a similar set of questions, but does so in the context of a model with a banking system.

Market games

To provide microeconomic foundations for monetary equilibrium, Shubik (1972) introduced a game that integrates the use of money as a medium of exchange with a generalized Nash–Cournot model of markets. The generalization by Shapley and Shubik (1977) can be summarized as follows. There is an exchange economy with \( \ell \) commodities, indexed by \( h = 1, \ldots, \ell \), and \( I \) traders, indexed by \( i = 1, \ldots, I \). Each trader is characterized by a consumption set \( R_+^I \), an endowment \( e_i \in R_+^I \), and a utility function \( u_i : R_+^I \to \mathbb{R} \). The utility functions are assumed to be \( C^1 \), non-decreasing and concave. We assume that each commodity has a positive aggregate endowment \( e_h > 0 \) and that each individual has a non-zero endowment \( e_i > 0 \).

For simplicity, we assume that traders offer their entire endowment of assets for sale and then bid for the assets they want to hold using fiat money as a means of payment. Each trader \( i \) has an endowment of fiat money \( m_i > 0 \).

A trader cannot bid more money than he holds, so the bid vector chosen by trader \( i \) must satisfy the cash-in-advance constraint

\[
\sum_{h=1}^{\ell} b_{ih} \leq m_i.
\]

The set of bid vectors satisfying the cash-in-advance constraint for trader \( i \) is denoted by \( B_i \), where it is understood that the initial balance \( m_i \) is exogenously given.

For any strategy profile \( b = (b_1, \ldots, b_I) \), define an attainable allocation of commodities as follows. Let the price of commodity \( h \) be denoted by \( p_h(b) \) and defined by

\[
p_h(b) = \frac{b_h}{e_h},
\]

where \( b_h = \sum_{i=1}^{I} b_{ih} \) and \( e_h = \sum_{i=1}^{I} e_{ih} \). Then let the quantity of commodity \( h \) received by trader \( i \) be denoted by \( \xi_{ih}(b) \) and defined by

\[
\xi_{ih}(b) = \begin{cases} 
    \frac{b_{ih}}{p_h} & \text{if } p_h > 0 \\
    0 & \text{if } p_h = 0.
\end{cases}
\]

Then the commodity bundle achieved by \( i \) for any strategy profile \( b \) is denoted by \( \xi_i(b) \). It is easy to see that the \( I \)-tuple \( \{ \xi_i(b) \} \) is an attainable allocation for any \( b \in B \).

The traders must return their initial balances of fiat money to the government at the end of the game. This means that trader \( i \) must end the trading period with at least \( m_i \) units of money. We assume that any choice of \( b_i \) resulting in end-of-period money balances that are lower than \( m_i \) will yield a payoff of \( -\infty \). The terminal balance for trader \( i \) equals his initial balance \( m_i \) minus the sum of his bids \( \sum_{h=1}^{\ell} b_{ih} \) plus the revenue from the sale of his initial portfolio \( p(b) \cdot e_i \). It is easy to show that the terminal balance satisfies
\[
m_i - \sum_{h=1}^{\ell} b_{ih} + p(b) \cdot e_i = m_i - p(b) \cdot (\zeta_i(b) - e_i),
\]

so the terminal constraint is satisfied if and only if \( p(b) \cdot (\zeta_i(b) - e_i) \leq 0 \). For any strategy profile \( b \), let trader \( i \)'s payoff be denoted by \( \pi_i(b) \) and defined by

\[
\pi_i(b) = \begin{cases} 
  u_i(\zeta_i(b)) & \text{if } p(b) \cdot (\zeta_i(b) - e_i) \leq 0, \\
  -\infty & \text{if } p(b) \cdot (\zeta_i(b) - e_i) > 0.
\end{cases}
\]

Shapley and Shubik (1977) demonstrate the existence of a Nash equilibrium for this game under the additional assumption that for each commodity \( h \) there are at least two individuals whose utility is increasing in that commodity. They also provide conditions under which the equilibrium allocation converges to a competitive equilibrium as the number of traders increases without bound.

**Concluding remarks**

As Joseph Ostroy wrote in the first edition of *The New Palgrave* (1987, p. 515),

> We shall argue that the incorporation of monetary exchange tests the limits of general equilibrium theory, exposing its implicitly centralized conception of trade and calling for more decentralized models of exchange.

That comment is just as true today as it was then, and remains the great challenge for economists who want to develop more satisfactory models of the process of monetary exchange at the level of the economy as a whole.

Douglas Gale

**See also**

- monetary approach to the balance of payments;
- monetary cranks;
- money illusion;
- monetary policy, history of;
- money supply.

**Bibliography**


money and general equilibrium


**Index terms**
cash-in-advance constraint
classical dichotomy
complete markets
excess demand
financial securities
incomplete markets
intertemporal substitution effect
money
money demand
money in general equilibrium
money supply
nominal prices
optimum quantity of money
Pigou effect
real balance effect
Say’s Law
stationary states
temporary equilibrium
uniform tightness property
value theory
Walras’s Law
wealth effect

**Index terms not found:**

incomplete markets
money in general equilibrium
value theory