Expected Stock Returns and the Correlation Risk Premium

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Simple question: Is the aggregate market return predictable?
  - Predictable out-of-sample?

Narrow focus: Forward-looking Variables - Option-Implied
  - Is there a theoretical link of the Variance risk premium (VRP) and correlation risk premium (CRP) to the equity risk premium?

Redundancy in variables or complementary?
  - CRP is a part of the market VRP!
Contribution and Major Results

1. Derive a market risk premium decomposition, including compensation for variance and correlation risk using a GE model.

   - Use contemporaneous instead of lagged predictive regression.
   - Use shocks to integrated variance/correlation under the $\mathbb{Q}$ measure.

   - VRP predicts best at quarterly horizon; then its performance weakens.
   - CRP weaker at short horizons, but predicts returns up to a year.

4. Study the sources for correlation risk premium.
Model Walkthrough

1. Use $N$ dividend trees adding up to an aggregate consumption process.

2. Variance of / and correlation between trees are stochastic.

3. $\Rightarrow$ Aggregate consumption variance is stochastic (2 components).

4. Solve for a GE assuming complete markets and EZ rep agent.

5. Derive market VRP and CRP between dividend claims.

6. Express the ERP as a function of VRP and CRP.
A large number of individual Lucas (1978) trees

- Variance is stochastic (a square root process)
- Correlation between trees is stochastic:

\[
\frac{dD_{i,t} \times dD_{j,t}}{\sqrt{(dD_{i,t})^2 \times (dD_{j,t})^2}} = \rho_{ij,t} dt.
\]

- Whereas all correlations are driven by the same state variable \( \rho_t \)
Adding up the trees produces the aggregate consumption process with stochastic variance:

\[
\begin{align*}
\frac{dC_t}{C_t} &= \mu_c dt + \delta_c \sqrt{V_t} dB_{c,t} \\
dV_t &= \kappa_1 (\bar{V} - V_t) dt + \sigma_1 \sqrt{V_t} dB_{V,t} + \sigma_\rho d\rho_t \\
d\rho_t &= \kappa_2 (\bar{\rho} - \rho_t) dt + \sigma_2 \nu (\rho_t) dB_{\rho,t},
\end{align*}
\]

Two-component variance:

1. Average individual variance shock: \( dB_V \)
2. Dividend correlation shock: \( d\rho \)

Preferences

- Representative investor with continuous-time, recursive preferences.
- \( J_t = E_t \left[ \int_t^T f(C_s, J_s) ds \right] \)
Obtain the pricing kernel dynamics:

\[
\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_1 dB_{c,t} - \lambda_2 dB_{\nu,t} - \lambda_3 dB_{\rho,t},
\]

where

\[
\begin{align*}
\lambda_1 &= \gamma \delta_c \sqrt{V_t} \\
\lambda_2 &= -\frac{1-\gamma \psi}{1-\gamma} A_1 \sigma_1 \sqrt{V_t} \\
\lambda_3 &= -\frac{1-\gamma \psi}{1-\gamma} (A_1 \sigma_\rho + A_2 \sigma_2) \sqrt{\rho_t}
\end{align*}
\]

: \( \lambda_2 \) — price of average individual variance risk

: \( \lambda_3 \) — price of correlation risk
Solve for the dynamics of the market

\[
\frac{dW_t}{W_t} = \zeta_W dt + \delta_c \sqrt{V_t} dB_{c,t} - A_{1a} dV_t - A_{2a} d\rho_t,
\]

Driven by shocks to consumption, its variance and dividend correlation

Derive the instantaneous variance \( V_W \) of the aggregate market and along these lines the VRP.
What about the CRP?

1. Given the individual dividend process

\[
\frac{dD_i,t}{D_i,t} = \mu_D dt + \sigma_D \sqrt{V_{i,t}} dB_{D_i,t} + \sigma_{DC} \sqrt{V_t} dB_{C,t}
\]

one can exploit the pricing kernel (and other identities / manipulations) to get the dynamics of a stock $S$.

2. This allows to calculate the variance of an individual stock, the correlation dynamics between any two stocks, and the CRP.
One can write the market risk premium as the sum of the three risk components:

$$E^{IP} \left[ \frac{dW}{W} \right] / dt - r_{f,t} = \lambda_1 \delta c \sqrt{V_t} + A_{1z} VRP_t + A_{2z} CRP_t,$$

- Both risk premiums are directly observable from options data!
- Decomposition is similar to Bollerslev, Tauchen, and Zhou (2009)
  We have one additional component – CRP

One can rewrite the market process in terms of the market variance and the correlation between stocks

$$\frac{dW_t}{W_t} = \zeta'_W dt + \delta c \sqrt{V_t} dB_{c,t} - A_{1z} dV_{W,t} - A_{2z} d\rho_{S,t} - A_{3z} dV_{i,t},$$
\[ r_{s \rightarrow s + \tau_r} = a + b \text{VRP}(s, s + \tau_r) + c \text{CRP}(s, s + \tau_r) + \epsilon, \]

<table>
<thead>
<tr>
<th></th>
<th>30d- Ret</th>
<th>91d- Ret</th>
<th>182d- Ret</th>
<th>273d- Ret</th>
<th>365d- Ret</th>
</tr>
</thead>
<tbody>
<tr>
<td>VRP</td>
<td>-0.322</td>
<td>-0.562</td>
<td>-0.304</td>
<td>-0.599</td>
<td>-0.740</td>
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<td>CRP</td>
<td>0.076</td>
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<td>0.381</td>
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</tbody>
</table>

\[ R^2 \]

- 2.48 \hspace{2cm} 6.90 \hspace{2cm} 7.26 \hspace{2cm} 6.90 \hspace{2cm} 5.43
- 6.08 \hspace{2cm} 0.15 \hspace{2cm} 9.87 \hspace{2cm} 0.81 \hspace{2cm} 0.85

- In-sample predictability for the **VRP at short horizon**.
  (Bollerslev, Tauchen, and Zhou (2009))

- In-sample predictability for the **CRP at longer horizon**.

- Indication for **non-redundant** information in the two variables.
Estimate the parameters of the pricing equation by running a rolling-window (last 3 years) regression for $t$:

$$r_{s \rightarrow s + \tau_r} = \alpha + \beta_{VRP} \ VRP(s, s + \tau_r) + \beta_{CRP} \ CRP(s, s + \tau_r),$$

where $s + \tau_r \leq t$, and regressions are estimated at $s$.

Regress historical market excess returns on lagged regressors,
We estimate the exposures using a contemporaneous regression:

\[
\frac{dW_t}{W_t} = \zeta'_W dt + \delta_c \sqrt{V_t} dB_{c,t} - A_{1z} dV_{W,t} - A_{2z} d\rho_{S,t} - A_{3z} dV_{i,t},
\]

relying on shocks to variance and correlation.

Innovation:
Quadratic co-variation depends **only** on stochastic parts
- which do not change by the change of measure (Girsanov).

\[
\frac{dW^{\mathbb{Q}}_t}{W_t} = \zeta''_W dt + \delta_c \sqrt{V_t} dB^{\mathbb{Q}}_{c,t} - A_{1z} dV^{\mathbb{Q}}_{W,t} - A_{2z} d\rho^{\mathbb{Q}}_{S,t} - A_{3z} dV^{\mathbb{Q}}_{i,t},
\]

→ Mix realized returns and shocks to option-implied variables!
Estimate the betas using shocks to IV and IC:

\[ r_{s+1} - r_{f,s} = \alpha + \beta_{t,\Delta IV} \Delta IV(s + 1, T) + \beta_{t,\Delta IC} \Delta IC(s + 1, T) \]

Daily increments in forward-looking moments (last year) —up to now.

Important: Adjust the betas for difference in variances of regressors and predictors:

\[ \beta_{t,VRP} = \beta_{t,\Delta IV} \times \frac{\text{Vol}(\Delta IV(t, t + \tau))}{\text{Vol}(VRP(t, t + \tau))}. \]

Form a market excess return forecast for horizon \( \tau_r \) using

- resulting betas (\( \beta_{t,VRP} \) and \( \beta_{t,CRP} \)), and
- time-\( t \) observable variables \( VRP(t, t + \tau_r) \) and \( CRP(t, t + \tau_r) \),
- no intercept is included in the forecast
Compare 4 models: historical mean, VRP, CRP, and VRP+CRP.

Performance Criteria

- **OOS $R^2$**
  \[
  R_{j,\tau_r}^2 = 1 - \frac{MSE_{j,\tau_r}}{MSE_{1,\tau_r}}, \quad \text{where} \quad MSE_{j,\tau_r} = \frac{1}{N} e_{j,\tau_r}^\top x e_{j,\tau_r}.
  \]

- Diebold and Mariano (1995) loss function:
  \[
  \delta_{j,\tau_r} = MSE_{j,\tau_r} - MSE_{1,\tau_r}.
  \]

- Gain in the Certainty Equivalent of a mean-variance (log) investor:
  \[
  \Delta CE_{j,\tau_r} = CE_{j,\tau_r} - CE_{1,\tau_r}, \quad \text{where} \quad CE_{j,\tau_r} = E[r_{j,\tau_r}^{MV}] - \frac{\gamma}{2} \sigma^2(r_{j,\tau_r}^{MV}).
  \]

<table>
<thead>
<tr>
<th>Days</th>
<th>VRP</th>
<th>CRP</th>
<th>VRP+CRP</th>
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<tbody>
<tr>
<td>30</td>
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<td>-0.014</td>
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<td>91</td>
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<tr>
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</tbody>
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- Practically, **no predictability** (adj. $R^2 < 0$, $\delta > 0$).
- **Common finding** for many variables (Goyal and Welch (2008)).
<table>
<thead>
<tr>
<th>Days</th>
<th>$R^2_{j,Tr}$</th>
<th>VRP</th>
<th>CRP</th>
<th>VRP+CRP</th>
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<tr>
<td>182</td>
<td>0.062</td>
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<td>0.091</td>
<td>0.001</td>
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</tbody>
</table>

- **Strong** out-of-sample predictability for **VRP** at short horizon.
- **Strong** out-of-sample predictability for **CRP** even at longer horizons.
Traditional Approach

- Very sensitive to outliers
- Delay of data in out-of-sample predictions.
- Example: Predict quarterly returns.
  - In the beta estimation, the last observation is $r_{t-3m \rightarrow t}$, and ... 
  - ... the most up-to-date RHS variables are 3 months old!

Contemporaneous Betas Approach

- Most recent RHS variables are from the day of the forecast.
- “High-Frequency” daily data (over the last year).
Equilibrium model confirms: both CRP and VRP predict the market risk premium.

Contemporaneous regression approach leads to strong OOS predictability (compared to the traditional OOS).

CRP works for longer horizons compared to VRP.

Different return predictability horizons due to different horizons in risk channels predictions.


