MASTER PROJECT

An Open-Economy Neoclassical Growth Model with a Two-Party System
An Extension of Aguiar and Amador (2010)

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Abstract

We construct an open-economy Neoclassical growth model with the particularity that two parties which have different views of the value of consumption when they are in office alternate in power. The base of the model is inspired in Aguiar and Amador (2010), but by introducing party heterogeneity on top of lack of commitment we are able to extend its insights and focus on the effects of the interaction between successive governments. We find that party heterogeneity gives raise to a pseudo steady state with a two-period cycle. Increased preference of parties for shifting consumption to their periods decreases the two steady-state values of capital compared to the first-best and to the case of a single party ruling forever. However, the more ‘spending’ party is able to enjoy higher capital and consumption levels than its opponent in the periods it is in office. Our model determines the dynamics of these two endogenous variables, as well as of taxes, transfers, foreign debt and trade balances for a given initial value of foreign debt.
## Contents

**1 Introduction**  
1.1 Related Literature .................................................. 3  
1.2 Aguiar and Amador (2010) .............................................. 4

**2 Environment**  
2.1 Domestic Firms .......................................................... 7  
2.2 Domestic Workers ....................................................... 8  
2.3 Domestic Government and Political Environment ................. 8  
2.4 The Equilibrium Concept ............................................... 11

**3 The Equilibrium Allocation**  
3.1 Problem and Optimality Conditions ................................. 13  
3.2 The Lagrangian Multiplier’s Dynamics ............................... 15  
3.3 Capital Dynamics ....................................................... 18  
3.4 Consumption Dynamics ................................................. 20  
3.5 Trade Surplus, Transfers and External Debt Dynamics ............ 23

**4 Simulation Analysis**  
4.1 Calibration ............................................................... 26  
4.2 Simulation Results ...................................................... 26

**5 Conclusion**  

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
</tr>
<tr>
<td>1.1</td>
<td>Related Literature</td>
</tr>
<tr>
<td>1.2</td>
<td>Aguiar and Amador (2010)</td>
</tr>
<tr>
<td>2</td>
<td>Environment</td>
</tr>
<tr>
<td>2.1</td>
<td>Domestic Firms</td>
</tr>
<tr>
<td>2.2</td>
<td>Domestic Workers</td>
</tr>
<tr>
<td>2.3</td>
<td>Domestic Government and Political Environment</td>
</tr>
<tr>
<td>2.4</td>
<td>The Equilibrium Concept</td>
</tr>
<tr>
<td>3</td>
<td>The Equilibrium Allocation</td>
</tr>
<tr>
<td>3.1</td>
<td>Problem and Optimality Conditions</td>
</tr>
<tr>
<td>3.2</td>
<td>The Lagrangian Multiplier’s Dynamics</td>
</tr>
<tr>
<td>3.3</td>
<td>Capital Dynamics</td>
</tr>
<tr>
<td>3.4</td>
<td>Consumption Dynamics</td>
</tr>
<tr>
<td>3.5</td>
<td>Trade Surplus, Transfers and External Debt Dynamics</td>
</tr>
<tr>
<td>4</td>
<td>Simulation Analysis</td>
</tr>
<tr>
<td>4.1</td>
<td>Calibration</td>
</tr>
<tr>
<td>4.2</td>
<td>Simulation Results</td>
</tr>
<tr>
<td>5</td>
<td>Conclusion</td>
</tr>
</tbody>
</table>
1 Introduction

This paper analyzes the interactions and macroeconomic policies pursued by two political parties in a small open economy. We propose an extension of the recent work by Aguiar and Amador (2010) (henceforth AA) to incorporate a two-party system. The political process is described in a simple deterministic fashion. Both parties alternate in office, one in even periods and the other one in odd periods. One motivation for this assumption is a country with bipartisan system that changes governments at a regular frequency. Alternatively, one could think of an economy dictated by a regime and subject to periodic interventions from outside institutions, such as the International Monetary Fund (IMF).

Two frictions prevail in our model economy: lack of commitment and political frictions. On the one hand, lack of commitment allows policymakers to depart ex-post from their announced tax rate on capital income and from repayment of sovereign debt. The degree to which the government can deviate is interpreted as a proxy for measures of institutional qualities, such as the respect for property rights. Indeed, we assume that the government can set the tax rate arbitrarily high, and therefore our model primarily addresses developing countries where risk of expropriation and debt repudiation is present.

On the other hand, political frictions arise from the political incumbent having a distorted view on intertemporal trade-offs. Every party strictly prefers consumption to occur while in office. In general, in the context of developing economies, this assumption can be motivated by a party absorbing a disproportionate share of public spending, for example through corruption. Another motivation is political disagreement between potential incumbents about the composition of government spending, as in Alesina and Tabellini (1990). Yet another interpretation is that incumbency enables political parties to direct government money to local projects in order to win votes.

Alesina (1987) highlights two different ways of modeling the relationship between the political system and macroeconomic policy. First, it can be assumed that the only objective of political parties is to gain and retain office. Essentially, this approach describes a political landscape in which parties compete for the median voter. Their policies matter only to the extent as they affect the electoral outcome. Alternatively, different political parties are
modeled as policymakers with different preferences.

This paper follows the second approach. In particular, we introduce two political parties sharing the same type of objective function, but different with respect to their preferred timing of consumption, as expressed by two distinct parameter values. One motivation for this assumption is that policymakers represent the interests of different constituencies. Two interest groups might have diverse tastes for the composition of the consumption good. Potentially, the government can influence this composition through transfer payments and thus each political incumbent will have different incentives to implement these transfers. That is, both parties differ concerning the degree of distortion of their view on the timing of consumption.

To put this in a more realistic context, we describe a bipartisan system of the following kind: two different political incumbents alternate periodically in office. One party adopts a policy that increases consumption levels during its term in office. Although the second party might also be inclined to conduct a macroeconomic policy that boosts consumption levels, it is more aware of its responsibilities that arise from incumbency and therefore has a weaker preference for consumption to occur while in power. Of course, we could also think of this second party being the above-mentioned international institution intervening the macroeconomic policy of the country in exchange for development aid, for example. The heterogeneity thus arising is modeled in a simple reduced-form way by specifying two different parameter values that describe the degree of political distortion. Henceforth, both political incumbents are therefore referred to as the ‘austere party’ and the ‘spending party’, respectively.

The main result of this paper is a deep understanding of the mechanisms at work in our model economy. We find that the most efficient way of boosting consumption levels, when the ‘spending party’ is in power, is a macroeconomic policy providing incentives for investments through low tax rates. A higher capital stock increases the worker’s marginal productivity and thus the wage. Additionally, the political incumbent is inclined to pay higher transfer payments. Overall, households therefore have more income in hand during periods where the ‘spending party’ is in power. On the other hand, high transfer payments and low tax revenues force the government to tap the international financial market and accumulate foreign debt.
As a result, the trade balance is worsened. The economy experiences a periodical cycle of two different macroeconomic policies inducing fluctuations. A realistic calibration of our model finds substantial support from empirical observations.

### 1.1 Related Literature

The analysis of interaction between two political parties with different preferences follows, for example, Alesina (1987). There, two policymakers have two different objective functions. Each party cares about inflation, but only one of them has also an incentive to create surprise inflation. For instance, the policymaker might want to reduce unemployment by exploiting a Phillips curve trade-off. With forward-looking rational agents, the party’s myopia results in the well-known inefficiency of time-inconsistent behavior. In particular, the economy experiences fluctuations in output and inflation linked to the political cycle. With credible commitment mechanisms, however, these macroeconomic fluctuations can be completely avoided if both parties agree on an intermediate policy.

Although our approach is conceptually very different, we find similar results. Particularly, our model also generates fluctuations in macroeconomic variables connected with the political cycle. Like Alesina (1987), we find that the economic fluctuations are wider the more polarized the political system is. Specifically, we shall show that the steady-state levels of macroeconomic aggregates depend on the degree of the political friction.

Alesina and Tabellini (1990) also analyze the interaction between political parties in the environment of a closed-economy Neoclassical growth model. In this paper, two political parties with different objective functions alternate in office. Political disagreement between them about the composition of government spending may generate a public debt level that tends to be larger than it is efficient. The more polarized the political landscape, the larger the public debt level at the steady state. However, because incumbents cannot default on their predecessor’s debt, lack of commitment does not pose problems. The key insight of their model is that public debt is used strategically by each government to influence the choices of its successors. Finally, Alesina and Tabellini (1990) argue that the quality of political institutions can help to explain the variance in debt policies pursued by different countries.

More generally, our model shows that institutional qualities, such as the respect for prop-
erty rights, can contribute to explain a substantial fraction of the fluctuations in macroeconomic variables. Regarding the prediction on public debt, our model yields that a positive amount of debt is sustained for a reasonable calibration. Moreover, the government can potentially affect the actions of its successor through the law of motion for public debt.

1.2 Aguiar and Amador (2010)

The present paper builds on the recent work by AA, which belongs to a strand of literature on the relationship between growth and the current account in developing countries. They analyze a small open economy in the environment of the Neoclassical growth model, and investigate its interactions with international financial markets. Only the government and capitalists have access to foreign borrowing and lending. Households supply labor and receive lump-sum transfers. Capitalists own and operate firms, which profits are taxed.

The extension we propose in the present paper adapts two frictions as introduced in AA. For completeness, we only restate the original assumptions here, and refer the reader to the previous subsections for their motivation. First, lack of commitment on the side of the political incumbent is assumed. Capital is sunk for one period after investment and tax policy can depart ex-post from its announcements and expropriate capitalists. A deviation on promised tax or debt payments forces the government into financial autarky. However, capitalists can still access international financial markets. Hence, the punishment is not only severe as the gains from international financial trade are lost, but also because capital flight to the international financial market decreases overall domestic investment.

Second, a political economy friction distorts the political incumbent’s view on intertemporal trade-offs: parties strictly prefer private consumption to occur while in power. The political process is stochastic, so that the electoral outcome is uncertain. This is one of the major differences to the present paper. AA show that the degree to which the view of the government is distorted has a negative effect on the economy’s speed of convergence to the steady state. The higher the value of incumbency to political parties, the more costly it is to move consumption away from a current incumbent. A low savings rate, in turn, prevents the economy from accumulating capital. At the same time, sovereign debt and capital are complementaries along the transition path. Because the temptation for the political incumbent
to default is stronger when capital is high, a rapid-growing developing economy accumulates net foreign asset positions to counteract this tension. This finding rationalizes the “allocation puzzle” of Gourinchas and Jeanne (2009). Particularly, it also takes account of the fact that the puzzle is mainly driven by governments accumulating foreign assets.

We summarize the key intuition of AA as follows. A political party in power is willing to accumulate external sovereign debt and impose ex-post high taxes on capitalists to increase contemporaneous private consumption. However, a large foreign sovereign debt position exacerbates the incumbent’s temptation to default and expropriate capitalists. In the face of these risks, capitalists desire to invest abroad. This negative effect of sovereign debt on future investment is also known as debt overhang effect. Eventually, a trade-off between debt overhang and economic growth prospects arises: too much foreign liabilities bring the domestic capital stock below its first-best level and thus hinder economic growth.

The remainder of this paper proceeds as follows. In Section 2 we present the environment of the model, including our extension with respect to AA. Section 3 features the derivation of the dynamics and the steady state of the equilibrium allocation for a linear utility function. Section 4 discusses a calibration of the model and contains a simulation that allows us to assess its empirical performance. Section 5 concludes. The Appendix shows the derivation of the equilibrium with concave utility, and a discussion on two benchmarks of our model: the cases of a one-party system and a social planner.

2 Environment

The environment of our model is closely based on AA, since we follow the same structure and many of our assumptions are replications of their paper. However, our extension to a two-party system implies significant differences in terms of the solution of the model, which allow us to provide new insights on the impacts of two-party political systems on economies that are open to international trade.

We use a small open economy with utility-maximizing, infinitely-lived agents that make intertemporal consumption decisions over a single good, which price we normalize to one. The economy is open in the sense that there exists an international financial market that
trades risk-free bonds with a fixed return \( R = 1 + r \) and in which capital flows freely. Firms operate on a perfectly competitive market and use two input technologies, labor and capital, which are paid out at market-clearing prices from the firm to the workers and capitalists, respectively. The government, on the other hand, taxes firms and makes transfers to working households. Furthermore, we allow for time-inconsistent behavior of the government by introducing the possibility of expropriation of capital and default on external debt.

The economy has a two-party system in which two different parties which only care about workers alternate in power. This change occurs with certainty, as we let one party rule in even periods and the other party rule in odd periods. We can therefore understand one period in our model as a political cycle. Hence, we assume two distinct cycles of the same duration. As a consequence, unlike in AA, the model is deterministic\(^1\). Nonetheless, we do maintain the assumption of the existence of an *incumbency effect*: while in power, parties care more about the utility of domestic workers than otherwise. This changes the view of the political incumbent on intertemporal trade-offs compared to the party in the opposition. In particular, we introduce an incentive for parties to incline the consumption path of the economy towards periods of incumbency. Heterogeneity across political agents enters by letting them differ in their correspondent incumbency factors.

This characterization is loose enough to be given various interpretations. For example, this could be the case of a severely indebted country which is subject to periodic interventions from an outside institution such as the IMF. In this set-up, the ‘local’ party would have a stronger desire to switch consumption to the periods when it is in power, while the ‘outside’ party (or international institution) would be more neutral – closer to the behavior of a benevolent social planner. Indeed, some indebted developing countries appear to be trapped in a pattern close to the one described by our model, where spending governments and externally imposed adjustment plans successively alternate, meaning that the assumption of a certain political process may not be that far from reality\(^2\). Alternatively, the heterogeneity we consider could

\(^1\)In AA, political parties have different probabilities to (re)gain office depending on whether they are already incumbents. This assumption is dropped here. However, we should bear in mind that all agents in the economy understand fully the equilibrium mechanism (namely, the deviation strategy, the punishment and the like, as defined later on) and that they form rational expectations.

\(^2\)See Easterly (2005)
be generated by the existence of one party favoring a minority (either ethnical, religious or regional) with special tastes about government transfers and another party serving a more representative fraction of the population. Again, this could for instance apply to ethnically fragmented developing countries, which tend to feature higher political and macroeconomic instability.\footnote{See Easterly and Levine (1997)}

\section{2.1 Domestic Firms}

The initial period is \( t = 0 \) and time discretely runs to infinity. Domestic firms are owned by capitalists and they all produce a single consumption good by using capital \( (k_t) \) and labor \( (h_t) \) according to a production function \( f(k_t, h_t) \), where \( h_t \leq 1 \). We assume \( f \) is homogeneous of degree one and satisfies the standard Inada conditions.

As in AA, capital is completely mobile internationally at the beginning of each period, but it is sunk for one period after investment occurs. Furthermore, it depreciates every period at a constant rate \( \delta \in [0, 1] \) and is rented out from abroad at an interest of \( r \). Workers supply labor to firms, which pay \( w_t \) as the wage. Both input prices are taken as given by the firm and they both clear their respective markets, which are perfectly competitive. Finally, the government taxes firm profits \( \pi_t \) at a rate \( \tau_t \in [0, 1] \), where \( \pi_t = f(k_t, h_t) - w_th_t \), and we denote \( \bar{\tau} > 0 \) as the government’s maximal tax rate.

The profit maximizing representative firm thus chooses inputs \( \{k_t, h_t\}_{t=0}^{\infty} \) such that

\begin{align}
(1 - \tau_t)f_k(k_t, h_t) &= r + \delta & (1) \\
f_h(k_t, h_t) &= w_t & (2)
\end{align}

Note that equation (1) is a non-arbitrage condition that states that the return of investing abroad (given by \( r \)) must equal the return of investing in the firm by using the capital in production (given by \( f'(k_t) - \delta \)). As we shall show in the next subsection, workers provide labor inelastically because they only care about consumption. Given their time endowment, this means that \( h_t = 1 \ \forall t \). We denote \( k_t^* \) as the capital satisfying equation (1) given no taxes \( (\tau_t = 0) \) and a mass one of labor, such that \( f_k(k_t^*) = r + \delta \), where \( f_k(k_t^*) \) is the derivative of...
\( f(k^*_t) \equiv f(k^*_t, 1) \) with respect to the first-best \( k_t \). It will become clear in our welfare analysis that \( k^*_t \) would be the capital chosen by a social planner without lack of commitment.

### 2.2 Domestic Workers

Domestic workers supply labor inelastically to the domestic firms. Labor is not traded internationally, meaning that workers do not participate in international markets.

We assume the existence of a representative working household that takes utility maximizing decisions on per capita consumption. This worker discounts future periods with an exponential discount factor of \( \beta \in (0, 1) \), and consumption levels \( \{c_t\}_{t=0}^{\infty} \) are determined at the initial period \( t = 0 \) in order to maximize the intertemporal utility given by

\[
U_0 \equiv \sum_{t=0}^{\infty} \beta^t u(c_t)
\]  

where \( u' > 0 \) and \( u'' \leq 0 \). Finally, workers receive lump-sum transfers \( T_t \) from the government. Hence, they can afford their consumption subject to a budget constraint \( c_t = w_t + T_t \).

### 2.3 Domestic Government and Political Environment

We consider a government which only cares about workers and whose only role is to choose the consumption level of households at every period by means of profit taxation and lump-sum transfers. The government receives revenues from taxing firm’s profits and issuing bonds internationally in order to pay for the transferred resources and the accumulated external debt. Consequently, the public budget constraint is

\[
\tau_t \pi_t + b_{t+1} = Rb_t + T_t
\]  

where \( b_t \) is external debt due at the beginning of period \( t \). Moreover, we assume that the level of taxes is non-negative and bounded from above by \( \tau_t \leq \bar{\tau} \ \forall t \). Although this assumption is needed in order to define the deviation strategy of a government which decides to expropriate capital, it could be interpreted as the maximum amount of rents that the government can extract from capitalists without being subject to a coup d’État.
Combining the budget constraints for the government and the workers, we can construct the aggregate resource constraint of the economy:

$$\tau_t \pi_t + b_{t+1} + w_t = c_t + R b_t$$  \hspace{1cm} (5)

At this point, we augment AA’s model as follows. We assume the existence of two political parties that alternate in office. One party is the incumbent in even periods while the other party is in power in odd periods. Because we do not allow for re-election to be an uncertain component, the model is deterministic in the sense that both political parties know that they will rule every two periods while being in the opposition the remaining ones.

Formally, let $I = \{E, O\}$ be the set of parties we define. We denote $i \in I$ as the ruling party, where $i = E$ indexes the party ruling in even periods and $i = O$ denotes the ruling party in odd periods. As in AA’s model, we assume that the incumbent party strictly prefers consumption to occur while in power. However, the introduction of two different parties in this political setting permits us to extend this assumption by allowing the parties to put different weights $\theta_i$ on the consumers’ utility. This is stated in the following key assumption:

**Assumption 1** (Incumbency Effect) Party $i = \{E, O\}$ enjoys utility $\theta_i u(c)$ while in power and utility $u(c)$ while in the opposition, where $\theta_i > 1 \ \forall i \in I$.

Note that $\theta_i$ measures how much the incumbent party $i \in I$ weights utility of consumers relative to the situation in which it is not in office, as in the latter this party attaches a weight to worker’s utility that we normalize to one. Furthermore, the $\theta$ parameter might differ across parties. From now on, we shall refer to the party with the higher $\theta$ as the ‘spending’ party, since in equilibrium it shifts consumption toward a higher immediate spending while in office, and to its opponent as the ‘austere’ party, because it lowers consumption in order to repay debts.

The payoff structure of the government is determined as follows. Let $P = \{0, 2, 4, 6, \ldots \}$
be the set of even numbers. The utility of the incumbent in even periods is:

\[ W_t^E = \theta_E \sum_{j=t}^{\infty} \beta^{j-t} u(c_j) + \sum_{j=t+1}^{\infty} \beta^{j-t} u(c_j) \]  

(6)

for \( t \in \mathbb{P} \), where \( \mathbb{N} \) denotes the set of natural numbers. Similarly, the incumbent ruling in odd periods has a utility

\[ W_t^O = \theta_O \sum_{j=t}^{\infty} \beta^{j-t} u(c_j) + \sum_{j=t+1}^{\infty} \beta^{j-t} u(c_j) \]  

(7)

for \( t \in \{\mathbb{N}\setminus\mathbb{P}\} \). Note in both cases that the first term of the sum captures the discounted infinite sum of utilities when the party is in office, whereas the second term is the discounted infinite sum of utilities when the party is in the opposition. This specification of parties’ utility reconciles with the idea that workers are in fact members of the political community, and as a consequence politicians are also maximizers of the utility that is derived from workers’ consumption in the periods when they are not in power.

Therefore, the payoff \( W_t \) of the incumbent party, and by extension the rest of the variables in the model, is defined for two states: even and odd periods. In particular

\[ W_t = \begin{cases} 
W_t^E & \text{for } t \in \mathbb{P} \\
W_t^O & \text{for } t \in \{\mathbb{N}\setminus\mathbb{P}\} 
\end{cases} \]  

(8)

Note that with this simple introduction of a two-party system in a deterministic manner we can no longer analyze problems of public commitment from the perspective of quasi-hyperbolic discounting, as in AA, because in our model the government is in fact an exponential discounter. However, we gain scope for new insights that AA’s model would not have been able to provide an answer to. For instance, we shall study how the parties interact in

\footnote{An even number is defined as a non-negative integer that is exactly divisible by two, that is, such that when dividing it by two the remainder is zero. This is the case for zero, and so \( \{0\} \in \mathbb{P} \). This is not trivial because it means that the starting ruling party is \( i = E \) for all possible parameterizations, which is potentially a crucial determinant for the dynamics of the model. However, as it will be clear later on, this does not have an important qualitative effect on the equilibrium path of the endogenous variables.}
the creation/repayment of debt, because the timing of our model (as seen in equation (4))
requires parties in office to repay the debt generated by the preceding political incumbents.

2.4 The Equilibrium Concept

The government is allowed to be time-inconsistent, as promises on tax policies or tax pay-
ments can be broken at any point in time. In order to recognize the existence of potential
incentives to deviate, we define a self-enforcing equilibrium where the incumbent government
receives a payoff $W(k_t)$. A self-enforcing equilibrium is sustainable whenever the equilibrium
levels of all endogenous variables ensure that the payoff of the party in office is weakly above
the deviation payoff $W(k)$. This is the so-called participation constraint of the government:

$$W(k_t) \leq W_t$$  \hspace{1cm} (9)

where $W_t$ is given by (8). The definition of the punishment strategy and deviation payoffs
is again inspired in AA: we assume that if either of the incumbent parties deviates from the
promised tax policies or debt payments, then international financial markets deny access to
the domestic government, which is therefore forced into financial autarky. After a deviation
occurs, the incumbent government will set taxes to the maximal rate $\bar{\tau}$ forever on, thus
triggering investment $k_t = k$ on the part of capitalists, where

$$(1 - \bar{\tau})f'(k) = r + \delta$$  \hspace{1cm} (10)

with $k = 0$ if $\bar{\tau} \geq 1$. Note that this after-deviation strategy constitutes an equilibrium,
because the government cannot set a tax rate that yields a better payoff (as $\tau_t \leq \bar{\tau}$ $\forall t$)
and the capitalists cannot be better off as they invest up to indifference. Moreover, it can
be shown that this deviation strategy is the harshest punishment as it implies the lowest
possible payoff for the incumbent party, that is, equation (9) holds for any self-enforcing
equilibrium. Loosely speaking, this is because deviation triggers autarky and thus the lowest
possible investment at home.

It remains to be shown how to express the deviation payoff $W(k_t)$. The deviation strategy
implies that if a government $i \in \Pi$ deviates, then it will get utility $\theta_i u(\bar{c}(k_t))$ in the period of
the deviation, where
\[ \bar{c}(k_t) = f(k_t) - (1 - \bar{\tau})f'(k_t)k_t \]  \hspace{1cm} (11)

In other words, when a deviation occurs, the government sets taxes \( \tau_t = \bar{\tau} \ \forall t \) but still enjoys a sunk capital \( k_t \) in the period of deviation, and so the domestic workers can enjoy the consumption level given by equation (11). Next period, the deviation is detected and the punishment strategy is triggered by condemning the government to no access to international financial markets, so that the government starts perceiving payoffs \( u(\bar{c}(k)) \) multiplied by the corresponding utility weight of the period from then on. Using equations (6) and (7), and by the formula of the infinite sum of a geometric series, it can be shown that the deviation payoff is

\[ W(k_t) = \theta_i u(\bar{c}(k_t)) + \beta \left( \frac{1 + \beta \theta_i}{1 - \beta} \right) u(\bar{c}(k)) \]  \hspace{1cm} (12)

for both \( i \in I \), with \( i = E \) if \( t \in P \) and \( i = O \) if \( t \in \{N \setminus P\} \).

Finally, the equilibrium needs to be feasible. Recall that the aggregate resource constraint at each period \( t \) is given by equation (5). The budget constraint requires that the present discounted value of future trade surpluses (the difference between output and domestic income, that is, capitalists’ rents plus workers consumption) is higher or equal than initial external debt. Given that the bonds are traded at the risk-free interest rate \( R \) and that the No-Ponzi-Game condition (NPGC) must hold, the dynamic budget constraint is

\[ b_0 \leq \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r + \delta)k_t - c_t) \]  \hspace{1cm} (13)

given some level of initial debt \( b_0 \). All in all, our definition of equilibrium is the following:

**Definition 1** A self-enforcing equilibrium is a sequence \( \{c_t, k_t, b_t, \tau_t, w_t\}_{t=0}^{\infty} \) in which (A) firms are maximizing profits; (B) workers are maximizing utility; (C) the labor market clears; (D) both the participation constraint (9) and the dynamic budget constraint (13) hold given an initial level of sovereign debt; (E) the government sets tax rates \( \tau_t \leq \bar{\tau} \ \forall t \).
3 The Equilibrium Allocation

3.1 Problem and Optimality Conditions

Definition 1 above describes how the model can be solved for the allocations of consumption, capital, debt, taxes and wages in equilibrium. The utility of the population is maximized for the path of endogenous variables in each period from $t = 0$ to $t = \infty$, given an initial stock of external debt $b_0$. We set the problem by taking into account the constraints of our political environment, i.e., by ensuring that each incumbent weakly prefers to follow the equilibrium path rather than to deviate from it. This solution method can be understood as a social planner which takes into account all the distortions present in the decentralized economy.

Therefore, the problem is

$$V(b_0) \equiv \max_{\{c_t,k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to:

$$b_0 \leq \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r + \delta)k_t - c_t)$$

$$W(k_t) \leq W_t$$

$$\bar{k} \leq k_t$$

$\forall t$ and given $b_0$. The constraint (17) implies that $\tau_t \leq \bar{\tau}$, which we assume not binding along the equilibrium path (that is, we assume $k_t > \bar{k} \; \forall t$) because the government never deviates.

We use a dynamic Lagrangian to solve the optimization problem. Let $\mu_0 > 0$ be the multiplier on the budget constraint (15), which is always binding. For convenience, we set $\lambda_t \mu_0 R^{-t}$ as the multiplier of the participation constraint (16) and $\phi_t R^{-t}$ as the multiplier of (17). Note that because the payoff $W_t$ of the government is defined differently depending on whether $t$ is odd or even, we need to derive first order conditions also distinguishing between odd and even periods.

On the one hand, the first order conditions for consumption in even periods ($t \in \mathbb{P}$) yield

$$\frac{1}{u'(c_t)} = \frac{(\beta R)^t}{\mu_0} + \lambda_0 \theta E(\beta R)^t + \lambda_1 (\beta R)^{t-1} + \lambda_2 \theta E(\beta R)^{t-2} + \cdots + \theta E \lambda_t$$

(18)
The multiplier in the initial period is therefore \( \lambda_0 = \theta_E^{-1}(1 - \mu_0^{-1}) \), where \( \mu_0 \) is given by the envelope condition \( V'(b_0) = -\mu_0 \) for a given \( b_0 \). The first order conditions for odd periods \((t \in \{N \setminus P\})\) can in turn be written as

\[
\frac{1}{u'(c_t)} = \left(\frac{\beta R}{\mu_0}\right)^t + \lambda_0(\beta R)^t + \lambda_1(\beta R)^{t-1} + \lambda_2(\beta R)^{t-2} + \cdots + \theta_O \lambda_t \tag{19}
\]

It is important to remark that the optimality condition for consumption, different to the case of the standard Neoclassical growth model, includes the multipliers of the participation constraints of all previous periods. This is because a raise in consumption today will increase the payoff of all previous governments and thus relax their participation constraints. Since all the terms corresponding to the previous and current participation constraints are positive, we can see that consumption will also be required to be higher than in the standard model for all \( t > 0 \).

The first order conditions for capital solve

\[
\lambda_t \theta_i = \frac{f'(k_t) - (r + \delta) + \phi_t}{u'(\bar{c}(k_t))\bar{c}'(k_t)} \tag{20}
\]

where \( \phi_t = 0 \) by the assumption \( k_t > k \ \forall t \), and \( i = E \) if \( t \in P \) and \( i = O \) otherwise. This condition implies that the ‘tax wedge’ in the capital market, \( f'(k_t) - (r + \delta) \), must be enough to compensate the incumbent government and avoid a deviation to \( \bar{\tau} \).

Next, we characterize the equilibrium dynamics of the variables in our model and their steady state. Later on we explain how the different parameters in our model, especially the incumbency factors \( \theta_E \) and \( \theta_O \), reflect into government behavior and the different equilibrium paths for the economy.

For simplicity, we shall assume throughout that utility is linear in consumption and that households are more impatient than the market. The former assumption is taken for simplicity but does not change results qualitatively, as it is shown in the Appendix, while the latter is necessary to prevent immediate convergence to the first-best level of capital.

**Assumption 2**  Assume \( \beta R < 1 \) and linear utility \( u(c_t) = c_t, \forall t \).
The description of the equilibrium proceeds as follows: first, we derive the dynamics of the multiplier directly from the first order conditions for consumption. These, together with the first order condition for capital, directly pin down the behavior of capital provided that in equilibrium the government does not deviate and the punishment strategy is not triggered. Long-run consumption follows by combining the participation constraint of the government, which binds in the steady state, with the steady-state levels of capital. Finally, external debt is obtained by using capital and consumption in the aggregate feasibility constraint of the economy.

3.2 The Lagrangian Multiplier’s Dynamics

First, let $t \in \mathbb{P}$. Note that the first order condition for consumption in even periods given by equation (18) can be rewritten as:

$$1 = \frac{(\beta R)^t}{\mu_0} + \sum_{j=0}^{t} (\beta RL)^j \lambda_t + \sum_{j=1}^{t-1} (\beta RL)^j \lambda_{t-1}$$

(21)

where $L$ represents the lag operator (i.e., $L^j \lambda_t \equiv \lambda_{t-j}$ for any $t, j$) and where $L^{t+s} \lambda_t = 0$ $\forall s > 0$. The last statement implies that we can make both sums in equation (21) run to infinity and use the formula of the infinite sum of a geometric series to solve for the multiplier. Doing so yields

$$\lambda_t = 1 - \frac{(\beta R)^2}{\theta_E} - \frac{\beta R}{\theta_E} \lambda_{t-1}$$

(22)

Now, let $t \in \{\mathbb{N} \setminus \mathbb{P}\}$. The first order condition for consumption in odd periods (19) can in turn be rewritten as:

$$1 = \frac{(\beta R)^t}{\mu_0} + \sum_{j=0}^{t} (\beta RL)^j \lambda_t + \sum_{j=1}^{t-1} (\beta RL)^j \lambda_{t-1}$$

(23)

Solving (23) for $\lambda_t$ leads to a similar expression as in the even-period case:

$$\lambda_t = 1 - \frac{(\beta R)^2}{\theta_O} - \frac{\beta R}{\theta_O} \lambda_{t-1}$$

(24)
The existence and uniqueness of a steady state can be proved as follows. Notice that if we combine equations (22) and (24) we can obtain

$$
\lambda_t = 1 - \frac{(\beta R)^2}{\theta_i} \left( 1 - \frac{\beta R}{\theta_{-i}} \right) + \frac{(\beta R)^2}{\theta_i \theta_{-i}} \lambda_{t-2}
$$

for all $i \in \mathbb{I}$ and $i \neq -i$. The reason why we use an expression for $\lambda_t$ as a function of $\lambda_{t-2}$ instead of $\lambda_{t-1}$ is that $t$ and $t-2$ are two adjacent periods in which the same government is ruling. Therefore, we need to capture the nature of the cycle by making explicit that, in the steady state, the $\lambda$ multiplier’s values will be the same only every two periods.

Note that, provided Assumption 2, the intercept of equation (25) is always positive and the slope $\frac{(\beta R)^2}{\theta_i \theta_{-i}}$ is lower than one. This means that $\lambda_t > 0$, $\forall t$ and, more importantly, that $\lambda_t = \lambda_{t-2}$ for some $t > 0$. Moreover, each multiplier converges monotonically to its corresponding steady state.

Figure 3.2 below plots equation (25) for both $i = \{E, O\}$ under the assumption $t \in \mathbb{P}$ and $\theta_O > \theta_E$ (other cases work similarly). Both graphs cross the 45° line, namely reach a steady state, for some $t > 0$, but the multiplier associated with the periods in which the incumbency effect is larger (in this case, the multiplier in odd periods) reaches a lower steady state. Furthermore, there exists convergence with a seesaw pattern such that $\lambda_t > \lambda_{t-1}$ $\forall t \in \mathbb{P}$ given the starting value $\lambda_0 > 0$. If we let the starting multiplier for even periods to be such that $\lambda_0 > \lambda_{E\infty}$, then the starting multiplier for odd periods is such that $\lambda_1 < \lambda_{O\infty}^\circ$. In this case, therefore, the multiplier in even periods converges monotonically from above towards its steady state, such that $\lambda_t > \lambda_{t+2}$ $\forall t < \infty$, whereas the multiplier in odd periods converges monotonically from below, such that $\lambda_{t+1} < \lambda_{t+3}$ $\forall t < \infty$, where we assumed throughout that $t \in \mathbb{P}$. This is reflected in Figure 3.2 where the dynamics for odd periods are displayed in red and the dynamics for even periods in blue.

\[5\] The latter statement can be shown by using equations (22)–(24) and the steady-state levels of the multipliers (which we provide later on) under the assumption $\lambda_0 > \lambda_{E\infty}^\circ$. 

16
By our reasoning above, solving for the actual steady state means imposing $\lambda_t = \lambda_{t-2} = \lambda_i^\infty$ in equation (25) for both $i = \{E, O\}$, where $\lambda_i^\infty$ denotes the steady-state level of the multiplier for periods in which party $i \in I$ is in office. The results are summarized in the following proposition:

**Proposition 1**  
*The value of the multiplier $\lambda$ in the steady state is*

$$\lambda_i^\infty = \frac{(1 - (\beta R)^2)(\theta_i - \beta R)}{\theta_i \theta_{-i} - (\beta R)^2}$$  \hspace{1cm} (26)

*for both $i = \{E, O\}$ and $i \neq -i$. Note $\lambda_i^\infty > 0 \ i \in I$ given the assumptions $\beta R < 1$ and $\theta_E, \theta_O > 1$.*

Hence, the economy converges to a pseudo steady state with a two-period cycle. The difference in the $\theta$’s fully explains the difference in the steady-state value of the multipliers: the stronger the incumbency effects differ across parties, the greater the jump between even and odd periods in the variables of the model in the long term.

We are interested in the behavior of $\theta_i \lambda_i^\infty$, since as it is seen in equation (20) this term will determine the level of capital in the steady state. First, it can be shown from equations...
and (24) that
\[ \frac{\partial \theta_i \lambda_i^\infty}{\partial \theta_i} > 0 \] (27)
for both \( i = \{E, O\} \). This result was already implied by equation (20): the stronger the incumbency effect, the lower the two capital levels in the steady state. This should be obvious, because increasing \( \theta_i \) for the government in power is equivalent to increasing its average impatience, which therefore leads to a lower level of capital in the steady state. However, what can be surprising is that the cross effect of an increase in \( \theta_i \) on the steady-state level of capital for the other party is higher than the direct effect on its own steady-state level of capital. In other words,
\[ \frac{\partial \theta_{-i} \lambda_{-i}^\infty}{\partial \theta_i} > \frac{\partial \theta_i \lambda_i^\infty}{\partial \theta_i} \] (28)
for both \( i = \{E, O\} \) and \( i \neq -i \). Equations (27) and (28) together imply that capital will be higher in periods when the party with a higher \( \theta_i \) is in power. This is one of the most important findings of our model and it is explained by the fact that raising capital levels, and therefore wages, is the most efficient way to increase the income of workers. So even if we could think a priori that the ‘spending’ party would set taxes higher than its political opponent, it turns out that it allows for increased investment in capital. This makes sense if we bear in mind that a social planner caring only about workers would set capital taxes to zero in the first-best. Hence, in this model taxes are only useful to repay debt, and it will be precisely the government with a lower \( \theta_i \) the one which will raise taxes in order to repay increased debt obligations and thus enjoy a lower level of capital at the steady state. In the first part of the Appendix we show that this result is robust to other specifications of utility.

3.3 Capital Dynamics

Next, we show how to derive the dynamics of capital from the first order condition (20). Our approach is close to AA’s. First, let us consider the element \( \lambda_i^\infty \theta_i \) for \( i = E, O \) and assume that:
Assumption 3 (Convexity) Let \( H(k_t) \equiv \frac{f(k_t)-\left(r+\delta\right)}{c'(k_t)} \). The function \( H(k) \) is strictly decreasing in \( k \) for all \( k \in [k, k^*] \).

Notice that, by equation (20), \( \lambda \theta_i = H(k_t) \) provided \( \phi = 0 \) \( \forall t \) by the assumption \( k_t > k \) and given that \( u'(\bar{c}(k_t)) = 1 \) by Assumption 2. Now, define \( K(\lambda_t) \) as the capital that satisfies equation (20):

\[
\lambda_t \theta_i = \frac{f'(K(\lambda_t)) - (r + \delta)}{c'(K(\lambda_t))} \tag{29}
\]

where (29) has a unique solution and \( K'(\lambda_t) < 0 \), both by Assumption 3. The dynamics of capital are

\[
k_t = \begin{cases} K(\lambda_t) & \text{for } \lambda_t \theta_i < \bar{\lambda} \theta_i \\ k & \text{otherwise} \end{cases} \tag{30}
\]

where recall that \( k \) is such that \( (1 - \bar{\tau})f'(k) = r + \delta \) (i.e., the after-deviation scenario) and where \( \bar{\lambda} \theta_i \) for \( i = E, O \) satisfies

\[
\bar{\lambda} \theta_i = \frac{\bar{\tau}(r + \delta)}{(1 - \bar{\tau})c'(k)} \tag{31}
\]

as equation (31) represents the first order condition of capital as in equation (20) when a deviation has occurred and the punishment strategy has been triggered. Namely, equation (31) corresponds to setting \( k_t = k \) to the first order condition of capital. Therefore, if \( \lambda_t \theta_i > \bar{\lambda} \theta_i \) for some \( t \) and both \( i = E, O \), then \( k^E_\infty = k^O_\infty = k \); as the punishment is maintained forever on.

In order to explicitly determine the path of capital when no deviation ever occurs (that is to say, the equilibrium path \( K(\lambda_t) \)), we need to further parametrize the model. In line with our calibration in the simulation section, we will assume a throughout Cobb-Douglas production function \( f(k_t, 1) = A_t k_t^\alpha \) with \( \alpha \in (0, 1) \) and \( A_t \) normalized to 1 without loss of generality. First, one can show that under this specification

\[
c'(k_t) = \alpha (\bar{\tau} + (1 - \bar{\tau})(1 - \alpha)) k_t^{\alpha-1} \tag{32}
\]
Using the last expression together with equation (29), we can obtain

\[ k_t = \left( \frac{\alpha}{r + \delta} \left( 1 - \left( \bar{\tau} + (1 - \bar{\tau})(1 - \alpha) \right) \lambda_t \theta_i \right) \right)^{\frac{1}{1 - \alpha}} \]  (33)

with \( i = E \) if \( t \in \mathbb{P} \) and \( i = O \) otherwise. Note that a steady-state level of capital is then straightforward to compute by simply imposing \( t = \infty \) and using equation (26) in the expression for the equilibrium dynamics of capital. This result is summarized in the next proposition.

**Proposition 2**  The value of capital in the steady state is

\[ k_i^\infty = \left( \frac{\alpha}{r + \delta} \left( 1 - \frac{(1 - (\beta R)^2)(\theta - \beta R)}{\theta \theta - \beta R} \theta \left( \bar{\tau} + (1 - \bar{\tau})(1 - \alpha) \right) \right) \right)^{\frac{1}{1 - \alpha}} \]  (34)

for both \( i = \{E, O\} \) and \( i \neq -i \), given a Cobb-Douglas production function \( f(k_i) = k_i^\alpha \).

We have already discussed the behavior of capital at the steady state with respect to changes in \( \theta_i \), which was completely determined by the participation constraint multiplier. Moreover, knowing the values of \( k_t \) for each period we can easily obtain the value of \( \tau_t \) from equation (1), and in particular note that periods in which capital is high will exhibit low capital taxation.

Furthermore, it is important to see that \( k_i^\infty > 0 \ \forall i \in \mathbb{I} \) given that \( \beta R < 1 \). The latter assumption is therefore crucial to find reasonable values for the variables in the equilibrium path. Otherwise (\( \beta R \geq 1 \)), we would have found negative steady-state values of the \( \lambda \) multiplier, implying that the participation constraint would not bind along the equilibrium path and that capital would have converged to \( k_i^* \) immediately. This last result relies on our assumption of non-negative taxes, but if one allowed for subsidies to capital it could be possible that the party with the higher \( \theta \) used them to set \( k_t > k_i^* \) and increase consumption for its periods along the transition to the steady state.

### 3.4 Consumption Dynamics

To our knowledge, there is no explicit way of deriving the exact dynamics of consumption. We propose approximating them by backing them out from the steady-state values via a recursive
method. This is an approximation because the steady-state values are never actually reached, but it is a sensible one since the economy approaches these values in a very small number of periods given realistic parameters, as it will be seen in Section 4. Let us first determine the two-period steady state and later on briefly explain the approximation technique.

First, note by equation (26) that \( \lambda^i_\infty > 0 \) for both \( i = \{E, O\} \). By the participation constraint (9), it is then clear that \( W^i_\infty = W(k^i_\infty) \), and using (12) evaluated at the steady state, then

\[
W^i_\infty = \frac{\theta_i c^i_\infty + \beta c^{-i}_\infty}{1 - \beta^2}
\]  

for both \( i = \{E, O\} \) and \( i \neq -i \). Using the last expression to look for the steady-state values, we obtain the results summarized in the following proposition.

**Proposition 3** The value of consumption in the steady state is

\[
c^i_\infty = \frac{\theta_i A^i - \beta A^{-i}}{\theta_E \theta_O - \beta^2}
\]  

with \( i = E \) if \( t \in P \) and \( i = O \) otherwise, and where

\[
A^i \equiv (1 - \beta^2) \left( \theta_i \bar{c}(k^i_\infty) + \frac{\beta(1 + \beta \theta_i)}{1 - \beta^2} \bar{c}(k) \right)
\]

for both \( i = \{E, O\} \). Note that \( c^i_\infty > 0 \) \( \forall i \in \mathbb{I} \) for reasonable parameterizations.

It can be shown that the effects on consumption of changes in the incumbency effect display the signs that we would expect:

\[
\frac{\partial c^i_\infty}{\partial \theta_i} > 0 \quad \frac{\partial c^{-i}_\infty}{\partial \theta_i} < 0 \quad \left| \frac{\partial c^i_\infty}{\partial \theta_i} \right| > \left| \frac{\partial c^{-i}_\infty}{\partial \theta_i} \right|
\]  

An increase in the incumbency effect for a given party raises consumption in the periods when this party is the incumbent while it hurts consumption in periods when its opponent is in office. Yet, the direct effect is now bigger than the cross effect. This means that parties with higher values of \( \theta_i \) will enjoy higher consumption levels in the steady state and that in general raising any \( \theta_i \) will raise the average level of consumption. However, as we shall show
in Section 4, this does not mean that social welfare improves. On the contrary, since under linear utility and impatient consumers the optimal strategy is to consume only in period 0, higher $\theta$’s set the economy further away from the first-best.

We can now use expression (36) to trace out the consumption dynamics from its two-period steady state. To do this, we first assume that the steady state of consumption (and, by extension, of all the other endogenous variables in the equilibrium path) is reached after a finite number of periods. In fact, in our simulations we find that the steady state is approached quite rapidly.

First, let $t$ be the period coming just before the steady state (i.e., suppose $t + 1 \equiv \infty$ in terms of the notation we have used so far). Moreover, suppose that party $i \in \mathbb{I}$ is ruling in period $t$. Then

$$W_i^t = \theta_i c_i^t + \left( \frac{\beta}{1 - \beta^2} \right) c_i^\infty + \left( \frac{\theta_i \beta^2}{1 - \beta^2} \right) c_i^\infty$$

Moreover, recall that $W_i^t = W(k_i^t)$ as $\lambda_i^t > 0 \ \forall t$, where $W(k_i^t)$ is defined as in equation (12) with $u(c(k)) = \bar{c}(k)$ under Assumption 2. Together with equation (39) this implies, if we solve for $c_i^t$, that

$$c_i^t = \bar{c}(k_i^t) + \frac{\beta}{\theta_i} \left( \frac{1 + \beta \theta_i}{1 - \beta^2} \right) \bar{c}(k) - B_1 \frac{\theta_i}{\theta_i}$$

where

$$B_1 \equiv \left( \frac{\beta}{1 - \beta^2} \right) c_i^\infty + \left( \frac{\theta_i \beta^2}{1 - \beta^2} \right) c_i^\infty$$

Similarly, for $t - 1$, we find an expression for $W_{t-1}^{-i}$ and use the fact that $W_{t-1}^{-i} = W(k_{t-1}^{-i})$ to solve for $c_{t-1}^{-i}$, which yields

$$c_{t-1}^{-i} = \bar{c}(k_{t-1}^{-i}) + \frac{\beta}{\theta_{-i}} \left( \left( \frac{1 + \beta \theta_{-i}}{1 - \beta^2} \right) \bar{c}(k) - c_i^t - B_2 \right)$$

where

$$B_2 \equiv \left( \frac{\theta_{-i} \beta}{1 - \beta^2} \right) c_i^\infty + \left( \frac{\beta^2}{1 - \beta^2} \right) c_i^\infty$$
and where $c_i^t$ is given by (40). Following the same procedure, in general for $t - j$ and $j = \{0, 1, 2, \ldots, t\}$ we find that

$$c_{t-j}^i = \bar{c}(k_{t-j}^i) + \frac{\beta}{\theta_i} \left( \frac{1 + \beta \theta_i}{1 - \beta^2} \right) \bar{c}(k) - \left( \sum_{k=2}^{j} \beta^k c_{t-j+k}^i + \frac{1}{\theta_i} \sum_{k=1}^{j-1} \beta^k c_{t-j+k}^{-i} \right) \frac{\beta B_1}{\theta_i} - \beta B_2 \theta_i^i$$

(44)

if in period $t - j$ the ruling party is $i$ (that is, if $j \in \mathbb{P}$). The sequence $c_{t-j}^{-i}$ will be defined similarly (with $B_2$ instead of $B_1$ in the last term) if in period $t - j$ the ruling party is $-i$ — that is, if $j \in \{\mathbb{N} \setminus \mathbb{P}\}$. Although equation (44) might not be useful to gain much economic intuition, we shall use it in our simulations to back out the consumption dynamics.

### 3.5 Trade Surplus, Transfers and External Debt Dynamics

Finally, we characterize the equilibrium dynamics of trade surplus, transfers and debt. Trade surplus is generated by government $i \in \mathbb{I}$ ruling in period $t$, and we denote it by $S_t^i$. In particular

$$S_t^i = f(k_t) - (r + \delta)k_t - c_t$$

(45)

that is, trade surplus corresponds to the part of the output that is not consumed privately nor used to pay the returns of the invested capital after depreciation. Notice that both the dynamics and the steady-state levels of trade surplus are trivially obtained by plugging those of capital and consumption in equation (45). In particular, provided a Cobb-Douglas production function

**Proposition 4** The value of trade surpluses in the steady state is

$$S_{\infty}^i = (k_{\infty}^i)^{\alpha} - (r + \delta)k_{\infty}^i - c_{\infty}^i$$

(46)

for both $i = \{E, O\}$, provided $f(k_t) = k_t^{\alpha}$. 

23
On the other hand, from equation (4) we can obtain an expression for the lump-sum transfers from the government to the households:

\[ T_t = \tau_t \pi_t + b_{t+1} - Rb_t \]  

(47)

where \( b_{t+1} - Rb_t \equiv -S^i_t \), that is, trade deficit. Using again a Cobb-Douglas production function \( f(k_t) = k_t^\alpha \) and by the optimality conditions of the firm, we can find that

\[ T_t = c_t + (r + \delta)k_t - (1 - \alpha \tau_t)k_t^\alpha \]  

(48)

where the dynamics of the tax rate \( \tau_t \) are obtained implicitly from equation (1). Thus, the dynamics and steady-state levels are directly pinned down by those of capital and consumption.

As for external debt, we can determine its steady state by evaluating the budget constraint (13) at the steady-state levels of \( k_t \) and \( c_t \) that we have found in the previous subsections. The two-period steady-state levels of debt are the sum of discounted future payments to financial markets, which must equal the sum of all future expected trade surpluses, that is

\[ b^i_\infty = \sum_{t=0}^{\infty} R^{-t} S^i_\infty + \sum_{t=1}^{\infty} R^{-t} S^{-i}_\infty \]  

(49)

The steady state of debt for each government depends upon the steady state trade surpluses of both political parties when they are in office. Given that \( S^i_\infty \) and \( S^{-i}_\infty \) are constant terms, we can take them out of the summation operator. The expression then reduces to the following value for the steady-state levels of debt:

**Proposition 5**  The value of external debt in the steady state is

\[ b^i_\infty = \frac{S^i_\infty + R^{-1} S^{-i}_\infty}{1 - R^{-2}} \]  

(50)

for both \( i = \{E, O\} \), where \( i \neq -i \).

In our simulations we show that, provided a reasonable calibration, the government sustains a positive level of external debt in the steady state, that is, has repayment obligations
towards other economies. Since the economy will converge to its steady-state level of debt for any possible exogenous initial debt level, the difference between the initial and the steady-state values will determine the behavior of all variables in their transition to the steady state. This can be seen by recalling that $\mu_0$ is determined by $b_0$. Namely, an economy starting with an initial amount of debt far from its steady-state level will present big spikes of capital and consumptions as parties alternate in government until debt adjusts to its steady-state value.

Once at the steady state, since we have shown that ‘spending’ governments set both lower taxes and higher consumption levels, it must be the case that they generate more debt, i.e., that the initial debt faced by the ‘austere’ government which comes next must be higher. In other words, a simple description of this economy at the steady state would be the following: ‘spending’ governments start their periods with low amounts of debt, which allows them to both pay positive transfers and set low taxes so that capital flows in and workers earn higher salaries, which they use to consume more (since they cannot access saving technologies). This lowers the trade surplus and therefore generates more debt for the next government. This subsequent government desires a flatter consumption path, since it is more indifferent across periods, so it will raise taxes and collect negative transfers to be able to repay the debt and avoid future capital being too low. Then, in this subsequent period both capital and consumption will be lower, and the trade surplus will be recovered.

In order to have a better interpretation of the results of our model and to see where the inefficiencies come from, in the Appendix we compare it with two different benchmarks. First, we study the case where there is a one-party system, i.e., the same party is in office for all the periods. Yet, we maintain the same assumptions regarding the lack of commitment, the deviation strategy and the equilibrium definition. Second, we turn to the analysis of the first-best allocation.

4 Simulation Analysis

It is possible to compute numerical solutions for the path of the endogenous variables of the model given its parameters. We first justify the values of these parameters and then analyze the behavior of the economy as it converges to its steady state, assessing the empirical
performance of the model.

4.1 Calibration

Our calibration of the main parameters of the model relies on the values used by AA, which correspond to the common assumptions in the Growth literature. There are however some specific modifications.

Firstly, the production function takes the standard Cobb-Douglas form

\[ y_t = A_t k_t^\alpha h_t^{1-\alpha}, \]

where the capital share is \( \alpha = \frac{1}{3} \), optimal labor is 1 and \( A_t \) is normalized to 1. The normalization is harmless since we are interested in variables as percentage of the first-best GDP. Given that political terms usually last four years, we set both the international real interest rate \( r \) and the depreciation rate \( \delta \) equal to 0.16. As it has been shown in Section 3, our model only displays interesting dynamics if households are more impatient than international markets. Nevertheless, since in the literature it is normally assumed that they are equally so, we set \( \beta = \frac{0.9}{R} \) to avoid moving away too much from the standard assumption.

Concerning the parameters that refer to the political environment, there are no empirical studies measuring the way governments perceive consumption of the population while they are in office compared to while they are at the opposition. Hence, we take the two extreme values used by AA, \( \theta_E = 3 \) and \( \theta_O = 7 \), to generate differences across the two political parties. The choice of these particular values is simply justified because they lead to realistic levels for the speed of convergence to the steady state in their model, so they can also be considered as a degree of freedom of our model to accommodate to different political environments. Finally, we let \( \bar{\tau} = 0.66 \) so that the average external-debt-to-GDP ratio of the two steady-state values is close to the median empirical value: about 28% of GDP.

4.2 Simulation Results

We analyze the case of linear utility of consumption\(^6\). Figure 4.2 shows the evolution of the economy as it converges to its two-period steady state for a given value of \( \lambda_0 \) (equivalent to an initial condition for external debt). It is important to recall that only the dynamics

\(^6\)The Appendix includes simulation results for the case of logarithmic utility
of capital are exact, while the values of consumption, trade surplus, transfers and debt are approximated by assuming that the steady state of these variables is actually reached in the last period of the simulation.

![Transition to Steady-State Cycle with Linear Utility](chart.png)

**Figure 4.2**: Simulation Results for Linear Utility

We can see how all the results which have been derived analytically in Section 3 are represented in Figure 4.2. Considering the first periods of the transition towards the steady state, where debt is far from its long-run level, we can appreciate the spikes in capital and consumption that are due to each government trying to counteract its predecessor in power. It should not be surprising that the economy approaches the steady state in such a low number of periods if we bear in mind that one period corresponds to four years. However, as in the standard growth model, the two steady-state values are never actually reached.

Once the economy becomes closer to the steady state, we are back to the stylized story of Section 3.4: ‘spending’ governments, which according to our calibration are in power in odd periods, set lower taxes and pay generous transfers, which allows both capital and consumption to be high (see dashed line). As a result, trade surplus falls and the next
government (see dotted line) steps in with a high debt to be repaid. In order to do so, it raises taxes and sets negative transfers, which depresses capital and consumption levels, but generates a positive trade surplus.

Regarding the levels of variables as a percentage of GDP, they all fluctuate around values consistent with empirical data\textsuperscript{[7]}. This is not surprising for the case of external debt, since we calibrated $\bar{\tau}$ such that this result was satisfied. However, the empirical performance of the model is also satisfying for other variables. The average level of consumption as a share of GDP in our empirical sample lies at 73%. The model’s predictions for consumption levels in steady state are 62% and 65%, respectively, and therefore approximately coincide with the empirical observations.

On the other hand, our model cannot qualitatively match an observed trade deficit of -10% on average in the sample. Instead, it predicts a trade surplus fluctuating between 3 and 5% of GDP at the steady state. Consistent with the model, this prevents the government to engage in a Ponzi scheme. That is, the cyclical trade surplus in steady state stabilizes the economy’s steady-state level of debt at a two-period cycle and prevents the country from piling up external debt.

Finally, public sector variables fluctuate at a much lower level than in real economies. This is in line with other studies incorporating the public sector as a maximizing agent, which all predict taxation and transfers levels to be much lower than the ones observed in reality. This can also be explained because in our model the government is not responsible for public goods provision, which in practice accounts for an important fraction of government

\textsuperscript{7}In fact, we compile a data set to compare empirical measures of consumption, external debt and trade surplus as a share of GDP, respectively, with our model’s predictions. Particularly, the sample contains panel data from Heston, Summers and Aten (2011) for PPP converted GDP per capita at constant 2005 prices and consumption as a share of the latter. Following AA, we create a measure of public net foreign assets as a share of GDP, defined as international reserves (excluding gold) minus public and publicly guaranteed external debt relative to GDP, all data in current US$ and collected from the World Development Indicators (WDI) provided by the World Bank. Lastly, we compute the current account’s share of GDP as the difference between exports and imports as a percentage of GDP, both from the WDI. The sample period spans from 1970 to 2004 and we restrict our focus to countries with an average PPP converted GDP per capita of less than $10,000 at constant 2005 prices, during that time, to account for the predominant weakness of political institutions in developing countries.
spending.

We do not provide estimates of the volatility of the variables in the steady-state two-period cycle because it is highly dependent on the values of $\theta_E$ and $\theta_O$, whose calibration is somewhat arbitrary, as it is argued earlier in this Section. An alternative way to proceed would be to choose these incumbency parameters such that the model replicates the empirical business cycle volatility, but this type of analysis goes beyond the scope of this study.

5 Conclusion

This paper extends a Neoclassical growth model with a small open economy and lack of public commitment, as presented in AA, by introducing a political system in which two parties, which differ on the weight they attach to private consumption while they are in office, alternate in power. Solving for the most efficient allocation given that the participation constraint of all governments is satisfied, i.e., given that no party has incentives to default and expropriate capital, we have shown that all variables converge to a two-period steady state.

Both the political distortion and lack of commitment drive the economy away from the first-best allocation, which in a context of impatience and linear utility corresponds to consuming all lifetime resources in $t = 0$. During the transition and at the steady state, the party with a stronger incumbency value manages to switch consumption to the periods while it is in office by lowering profit taxes such that the capital level increases. The other party is then forced to pay for the debt generated by setting negative transfers and raising profit taxes. Furthermore, as we show in the Appendix, all these results apply both to the cases of a linear and a logarithmic utility function.

It is important to remark that our predictions would have been substantially different if we had considered a closed economy where capital had to be accumulated through internal savings. The fact that in our economy capital could be immediately set to its first-best by simply lowering taxes and letting it flow in from abroad accounts for the high volatility we observed. It would be interesting to study how the economy would react in a set-up with partial financial liberalization or with adjustment costs to investment. We also believe that
further research could be conducted by devising more realistic ways of introducing party heterogeneity in Growth and Business Cycle models. For example, parties could differ in their support to (qualitatively) different groups of interest, in their preferences for different social layers or in their degree of knowledge about the state of the economy. In general, this field of study will be relevant to understand the behavior of economies in a context of political instability or foreign political intervention.
Appendix

This appendix presents the extension of the self-enforcing equilibrium to the case of non-linear utility, which we use in our simulation analysis, where we provide a brief overview of how to derive the equilibrium conditions of the main variables of the model. The second part of the Appendix includes the discussion of two benchmarks of our model: the case of a one-party system, in which the economy is ruled by a single incumbent for all time periods, and the discussion for the economy of the social planner, which yields the efficient allocations.

Appendix 1: Derivation of the Model under Non-Linear Utility

We briefly extend Assumption 2 to a case in which the utility function of workers is strictly concave. In this appendix we explain the method to characterize the steady-state values under this more realistic specification of utility. Since it is not feasible to derive comparative statics analytically for this case, we provide numerical solutions to show that the main results obtained with a linear utility function are preserved.

For the sake of simplicity, we shall use a logarithmic functional form.

Assumption 2’ Assume $\beta R < 1$ and a logarithmic utility $u(c_t) = \log c_t$.

The first order conditions for consumption as expressed in equations (18)-(19) can be written as

$$c_t = \frac{(\beta R)^t}{\mu_0} + \theta_E \sum_{j=0}^{t} (\beta RL)^j \lambda_t + \sum_{j=1}^{t-1} (\beta RL)^j \lambda_t$$ (51)

where $t \in P$ and where $L$ represents the lag operator. For odd periods, the first order condition for consumption is similarly defined. Solving each equation for the period $t$ multiplier by rearranging terms we obtain

$$\lambda_t = \theta_i^{-1} (c_t - (\beta R)^2 c_{t-2} - \beta R \lambda_{t-1})$$ (52)

with $i = E$ if $t \in P$ and $i = O$ whenever $t \in \{N \setminus P\}$. To find the steady-state level of
the multiplier, we proceed as in the linear case. Consider first the case \( t \in \mathbb{P} \). We plug in equation (52) its symmetric version for \( \lambda_{t-1} \), where here \( (t-1) \in \{\mathbb{N} \setminus \mathbb{P}\} \). Next, we impose the two-period steady state by letting \( c_t = c_{t-2} = c^E_t \), \( c_{t-1} = c_{t-3} = c^O_t \) and \( \lambda_t = \lambda_{t-2} = \lambda^E \) in the resulting equation. Rearranging and multiplying terms through, we reach the result summarized in the following proposition:

**Proposition 1’**  
*The value of the multiplier \( \lambda \) in the steady state is*

\[
\lambda^i_t = \frac{(1 - (\beta R)^2)(\theta - i c^i_\infty - \beta R c^{-i}_\infty)}{\theta_i \theta - i - (\beta R)^2}
\]  
*(53)*

*for both \( i = \{E, O\} \) and \( i \neq -i \), where the terms \( c^i_\infty \) and \( c^{-i}_\infty \) are the steady-state levels of consumption.*

The steady-state values of the rest of the variables are obtained similarly to the linear case. First, use again equation (20) and the fact that \( \bar{c}(k_t) = f(k_t) - (1 - \bar{\tau})f'(k_t)k_t \) and \( \bar{c}'(k_t) = \bar{\tau}f'(k_t) - (1 - \bar{\tau})f''(k_t)k_t \) to obtain that

\[
\lambda_t \theta_i = \frac{(f'(k_t) - (r + \delta))(f(k_t) - (1 - \bar{\tau})f'(k_t)k_t)}{\bar{\tau}f'(k_t) - (1 - \bar{\tau})f''(k_t)k_t}
\]  
*(54)*

with \( i = E \) if \( t \in \mathbb{P} \). Using a Cobb-Douglas production function \( f(k_t) = k_t^\alpha \) with \( \alpha \in (0, 1) \), the last expression reduces to

\[
\lambda_t = \theta_i^{-1}\left( k_t^{\alpha} - \frac{r + \delta}{\alpha} k_t \right)
\]  
*(55)*

On the other hand, in the steady state we know that \( W(k_{i\infty}) = W^i_{\infty} \) holds\(^8\) meaning

\[
\theta_i \log (\bar{c}(k^i_{\infty})) + \beta \left( \frac{1 + \beta \theta_i}{1 - \beta^2} \right) \log (\bar{c}(k)) = \frac{\theta_i \log c^i_\infty + \beta \log c^{-i}_\infty}{1 - \beta^2}
\]  
*(56)*

Next, combining equations (52) and (55) evaluated at the steady state we find that

\[
c^i_\infty = \left( \frac{\theta E \theta O (\beta R)^2}{\theta E \theta O (1 - (\beta R)^2)} \right) \left( (k^i_\infty)^\alpha - \frac{r + \delta}{\alpha} k^i_\infty \right) + \frac{\beta R}{\theta_{-i}} c^{-i}_\infty
\]  
*(57)*

\(^8\)In particular, this is true only if \( \lambda^i_\infty > 0 \) for both \( i = \{E, O\} \), by the participation constraint (9). Notice that from the steady-state levels of the multiplier in Proposition 1’, \( \lambda^i_\infty > 0 \) holds only if \( \theta_i c^{-i}_\infty > \beta R c^{-i}_\infty \), \( \forall i \in \mathbb{I} \) and \( \beta R < 1 \), which is true provided our calibration.
and using $c_\infty^i$ from a symmetric version of equation (57) into $c_\infty^i$, we conclude that

**Proposition 2’** The value of consumption in the steady state is a function of capital in the steady state, given by

$$
c_\infty^i = \left( \frac{(k_\infty^i)^\alpha - r^\alpha_k k_\infty^i}{1 - (\beta R)^2} \right) + \left( \frac{\beta R}{\theta_i} \right) \left( \frac{(k_\infty^{-i})^\alpha - r^\alpha_k k_\infty^{-i}}{1 - (\beta R)^2} \right)
$$

where $i = E$ if $t \in \mathbb{P}$.

Finally, combining (58) with (56), we find an implicit expression for the two steady-state levels of capital, which we summarize in the following proposition.

**Proposition 3’** The value of capital in the steady state is implicitly defined by

$$
\theta_i \log \left( (1 - (1 - \bar{\tau})^\alpha) (k_\infty^i)^\alpha \right) + \beta \left( \frac{1 + \beta \theta_i}{1 - \beta^2} \right) \log (c(k)) = \frac{\theta_i \log c_\infty^i + \beta \log c_\infty^{-i}}{1 - \beta^2}
$$

for both $i = \{E, O\}$ and $i \neq -i$, where $c_\infty^i$ is given in equation (58).

In our simulation, we use equation (59) for $i = E$ and $i = O$ to numerically solve for a fixed point in each case by setting the difference between the left-hand side and the right-hand side of equation (59) equal zero. Since each of these two equations are in terms of both $k_\infty^E$ and $k_\infty^O$, we make the guess that one steady-state value is proportional to the other, that is $k_\infty^i = \eta k_\infty^{-i}$ for an initial arbitrary $\eta \in \mathbb{R}$. This way we can iteratively solve for the roots of both equations by updating the value of $\eta$ until both roots become sufficiently similar.

We shall not characterize the steady-state levels of the rest of the variables for the logarithmic utility case, but it should be noted that these are straightforward to obtain: we can pin down the steady-state levels of consumption by plugging $k_\infty^E$ and $k_\infty^O$ into equation (58) for each $i$. Similarly, the two-period steady state for external debt can be found by using the expressions for capital and consumption in the steady state into a long-run version of the dynamic budget constraint (13). This would conclude the characterization of the long-run economy in the case of logarithmic utility.
Figure A.1 below provides simulation results for the two-party system economy with logarithmic utility and the same parameters as in Section 4. Although the analytical determination of the dynamics of the model becomes intractable in this case, which makes it impossible to provide economic justification for the behavior of the economy, we can compare the new steady state cycle that generates with its counterpart in the linear case. The figure shows that the introduction of logarithmic utility does not change the main qualitative results: the steady state behavior and levels of all variables are very similar. However, we can see that the volatility of capital is reduced and that of consumption is increased, which remains somewhat puzzling given that logarithmic utility introduces a smoothing motive for consumption.

![Steady-State Cycle with Logarithmic Utility](image)

**Figure A.1:** Simulation Results for a Logarithmic Utility

**Appendix 2: Benchmarks of the Model**

In this section of the Appendix we compare our model with two different benchmark cases. First, we study the case where there is a one-party system, i.e., the same party is in office for
all the periods. Yet, we maintain the same assumptions regarding the lack of commitment, the deviation strategy and the equilibrium definition. Second, we turn to the analysis of the first-best allocation by solving the social planner’s problem.

I. The One-Party System Allocation

First, we shall analyze the situation in which there is only one party with lack of commitment ruling in all time periods. This allows us to contrast the cyclical fluctuations that arise in a two-party system with the situation where the country is ruled by a dictator “for life”. Therefore, we are able to identify the inefficiency related to the political cycle.

In particular, we adapt Assumption 1 such that the party in power receives utility \( \theta_i u(c) \) and acknowledge that \( \theta_i = \theta \). However, precisely because this party is the only possible ruling party, the presence of \( \theta \) plays an irrelevant role, and without loss of generality we can set \( \theta = 1 \).

Because the government still has lack of commitment, the definition of an equilibrium as in Section 2 remains unaltered. Moreover, note that the problem considered here is exactly the same as that of Section 3 for the case of \( \theta_E = \theta_O = 1 \). Hence, our original model nests the case of a one-party political system and we can recover all equilibrium values by imposing these parameter values. If we proceed in this manner, we obtain that the ruling party chooses the consumption level in period \( t = 0 \) such that the economy reaches the steady state level in period \( t = 1 \). Hence, transitional dynamics only last for one period, and the equilibrium values for capital and consumption are

\[
k_t = \left( \frac{\alpha}{r + \delta} (\beta R + \alpha (1 - \bar{\tau})(1 - \beta R)) \right)^{\frac{1}{1-\alpha}}
\]

This can be rationalized in at least two ways. First, notice that \( \theta \) is a parameter that is associated multiplicatively with utility in all time periods. By the result that any linear combination of a utility function represents the same set of preferences because the ordering does not get altered, the presence of \( \theta \) is innocuous. Second, in the two-party case, \( \theta \) can be interpreted as the relative weight that the government attaches to private consumption relative to the weight attached in periods of non-incumbency. However, in this model the party is always in power and never in the opposition, which means that the political party does not attach any value to non-incumbency at all as this state of the world is not defined. Hence, \( \theta \) can be harmlessly normalized to one.
\[ c_t = (1 - \beta)c(k_t) + \beta\bar{c}(k) \]  \hspace{1cm} (61)

for \( t > 0 \), and

\[ k_0 = \left( \frac{\alpha}{r + \delta} (\lambda_0 + \alpha(1 - \bar{\tau})(1 - \beta R)) \right)^{\frac{1}{1 - \alpha}} \]  \hspace{1cm} (62)

\[ c_0 = \bar{c}(k_0) + \beta\bar{c}(k) - \beta\bar{c}(k_{t>0}) \]  \hspace{1cm} (63)

for the initial period \( t = 0 \), where \( \bar{c}(k_t) = ((1 - \alpha) + \alpha\bar{\tau})k_t \), \( \lambda_0 = 1 - \mu_0^{-1} \) and \( \mu_0 \) is given by the initial level of debt, \( b_0 \).

Since we are back to the world of our original models, we can use the comparative statics in Subsections 3.3 and 3.4 (see results (27), (28) and (38) above) to show that an environment with only one party (lower and equal \( \theta \)'s) will feature a higher capital stock and lower consumption levels at the steady state, which in this case is reached in one period. However, having a lower steady-state level of consumption does not imply that workers utility is lower, since this will allow the economy to devote more resources to consumption in period \( t = 0 \). As we shall show next, this consumption path is actually closer to the first-best, so it represents a welfare improvement with respect to the economy with two parties. Therefore, we conclude that disagreement among political parties is a source of inefficiency that drives the economy further from its welfare maximizing allocation.

II. The First-Best Allocation

Next we consider an economy where all sources of inefficiencies are absent. The planner has no lack of commitment and weights all periods in the same way as workers do. Hence, this problem does not include the self-enforcing equilibrium definition nor the deviation strategy of the market, as resources are allocated only by means of the feasibility constraint of the economy, thus without considering taxation revenues (as it will become clear below). In short, we do not consider political frictions, that is, Assumption 1 ceases to hold, and the problem does not include a participation constraint in the spirit of equation (16). Consequently, the cyclical nature of the equilibrium dynamics is no longer present.
Hence, we solve

\[
V(b_0) \equiv \max_{\{c_t,k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

subject to:

\[
b_{t+1} = Rb_t + c_t + (r + \delta)k_t - f(k_t)
\]

We maintain the same assumptions about the parameters and functional forms, meaning that utility is linear and consumers are more impatient than international markets ($\beta R < 1$). In this case, the dynamics of our endogenous variables are straightforward.

First, the planner allocates capital according to the non-arbitrage condition

\[
f'(k_t) = r + \delta
\]

That is, according to our previous notation, the efficient allocation of capital is $k_t = k_t^\ast$. In other words, the efficient level of capital coincides with that of the two-party system when the government sets $\tau_t = 0$, which is never the case along the self-enforcing equilibrium. This shows that the market equilibrium’s inefficiency partly arises from proportional taxation. Moreover, the planner allocates capital so that it remains constant over time.

On the other hand, if $u(c_t) = c_t$ and the representative household is more impatient than the market, then the social planner moves all available resources to the initial period. Hence, the household consumes at $t = 0$ the discounted sum of all lifetime wealth, and spends the rest of his life repaying the debt generated. Given that equation (13) binds due to a transversality condition (TVC)\footnote{Recall that equation (13) was expressed with an inequality by the NPGC. The TVC further states the result that not only the consumer cannot have debt in the limit (as assumed by the NPGC), but he also cannot leave any wealth unspent. The reason is that if this were the case, then he could be able to increase his level of utility by actually spending the remainder of this “last-period” wealth. The TVC together with the NPGC therefore imply that equation (13) holds with an equality. We use this fact to derive an expression for $c_0$.}, then the initial level of consumption is

\[
c_0 = \sum_{t=0}^{\infty} R^{-t} (f(k_t) - (r + \delta)k_t) - b_0
\]
given $b_0$, while $c_t = 0 \forall t > 0$.

Our definition of equilibrium was based upon the existence of lack of commitment—the risk of expropriation of capital and sovereign debt repudiation—, and political frictions—represented by two parties with different interests. This generated an inefficient equilibrium allocation due to the use of distortionary instruments (profit taxation) to satisfy the participation constraint. Comparing to the case of a one-party system, which is the best of all possible cases in the original model, we can see that the key difference is the absence of the participation constraint. Since it is no longer necessary to ensure that future governments enjoy enough consumption while in power, all consumption can be efficiently transferred to period $t = 0$ (as consumers are impatient), which allows for improved welfare levels.

To sum up, we have identified the two inefficiencies present in our original model by eliminating political discrepancies in the case of a one-party system and by also eliminating lack of commitment in the case of the social planner.
References


