Firm Dynamics and Pricing under Customer Capital Accumulation

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Motivation

- **Established Fact:**
  - Large degree of heterogeneity in firm revenue.
  - **Traditional view** → **Productivity differences** across firms.

- **New Fact:**
  - Variation in revenue largely explained by price differences.
  - Observed: within narrowly-defined industries; across sectors, across countries.
  - Smaller sellers set persistently lower prices, and take time to catch up.
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Questions:

1. Why do firms of similar productivity *price differently* within the same market?

2. What are the implications?
   - Micro level $\rightarrow$ Heterogeneity in life-cycle of firms.
   - Macro level $\rightarrow$ Aggregate dynamics of price markups.

Build and calibrate new model of demand-driven firm dynamics.
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What We Do

- **Model:**
  - **Blocks:** (i) Search and Matching in product market; (ii) Firm dynamics.
  - **Idea:** Sellers use *prices* to attract new customers $\Rightarrow$ *Endogenous markups.*
    - Dynamic trade-off: Profits now vs. market share later on.

- **Estimation:**
  - Augment model with aggregate supply/demand shocks.
  - Calibrate to micro-pricing data from the U.S.
    - **Key moments** $\rightarrow$ (i) Price and seller growth correlation; (ii) Average markup.
    - **Validation** $\rightarrow$ Untargeted moments from distribution of price changes.
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Preview of Results

**Theoretical:**

1. **Key Result**  ⇒  Smaller sellers set lower prices  →  Grow faster.

2. **Efficiency**  ⇒  Prices play both *distributive* and *allocative* role.

**Quantitative:**

1. **Response to aggregate shocks:**
   - Average effects  →  Procyclical markups, pass-through is incomplete.
   - Heterogeneous effects  →  Reaction stronger and more persistent for smaller firms.

2. **Secular trends**  →  1. Rise in average markup  [DeLoecker & Eeckhout ('17)]
   2. Decline in firm entry  [Pugsley & Sahin ('15)]

   Decline in buyer search cost  →  Generates about 25% of the long-run changes.
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Related Literature

1. **Customer markets in Macro:**
   Phelps and Winter (‘70), Rotemberg and Woodford (‘91, ‘99), Ravn, Schmitt-Grohé and Uribe (‘06), Nakamura and Steinsson (‘11), Gourio and Rudanko (‘14), Paciello, Pozzi and Trachter (‘17), Gilchrist, Schoenle, Sim and Zakrajsek (‘16).

   **Contributions:** (i) Micro-foundation; (ii) Firms life-cycle; (iii) Aggregate dynamics.

2. **Heterogeneity and firms’ life-cycle:**
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   **Contribution:** Quantitative model to rationalize empirical findings.

3. **Search-and-Matching models with firm dynamics:**
   Menzio and Shi (‘10), Elsby and Michaels (‘13), Kaas and Kircher (‘15), Schaal (‘17).

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Outline

1. Baseline Model
2. Data and Calibration
3. Quantitative Results
   1. Response to Aggregate Shocks
   2. Secular Trends in Markups
Environment

- Continuous and infinite time.
- Single homogeneous, indivisible, non-storable good.
- Two types of agents → Buyers ($B$) and Sellers ($S$).

- Frictional product market → Directed search.
  - Seller announces price for its products.
  - Buyers search across sellers, choose one.
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Agents

1. **Buyers** – Measure-one *(exogenous).*
   - **Unmatched** → Search for seller at cost $c > 0$.
   - **Matched** → Utility $v > 0$, pay price $p$.

2. **Incumbent sellers** – Measure $S > 0$ *(endogenous).*
   - **Typical seller** → $n \in \{1, 2, 3, \ldots\}$ customers *(seller’s size).*
   - **Separations** → $\delta_f > 0$ (seller dies); $\delta_c > 0$ (a customer leaves).
   - *(Linear) Technology:*
     - Sell one unit per customer → $y(n) = n$
     - Operating variable cost → $C(n)$, with $C'' > 0 = C''$

3. **Entrant sellers:**
   - Pay "market penetration" cost $\kappa > 0$ to reach $1^{st}$ customer.
   - *Free entry* into this market.
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Markets and Prices

- Seller posts price trajectory \((p_j : j \geq t)\) for match at time \(t\).

- Markets:
  - \(x \equiv \) Buyer’s expected value for given price path.
  - Tightness \(\rightarrow \theta(x) := B(x)/S(x)\).
  - Poisson meeting rates \(\rightarrow \eta(\theta)\) for seller; \(\mu(\theta)\) for buyer.

- Prices:
  1. Commitment \(\rightarrow\) Seller commits (e.g. reputation); Buyers may leave (at cost \(c\)).
  2. Anonymity \(\rightarrow\) All customers charged the same (no discrimination).

- Focus on symmetric Markov Perfect Equilibrium:
  
  \[ \text{Seller chooses} \rightarrow \omega = \{p, x'(n')\} \]

  where

  \(p\) : price charged to each customer.

  \(x'(n')\) : vector of promised utilities, for all \(n' \in \{n-1, n+1\}\).
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HJB Equations I: Unmatched Buyers

- Directed search \( U^B = \max_x u^B(x) \), where:
  \[
  ru^B(x) = -c + \mu(\theta(x))(x - u^B(x))
  \]
  \( r \) is the search cost, \( \mu(\theta(x)) \) is the expected match value.

- Indifference → Active markets make buyers ex-ante indifferent:
  \[ u^B(x) \leq U^B, \quad \text{with equality if } \theta(x) > 0 \]  
  \([\text{IC}]\)

- Pins down:
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- Search cost

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1. Equilibrium mapping \( \theta : x \mapsto \mathbb{R}_+ \), increasing in \( x \).
2. Set of equilibrium markets \( \Rightarrow X^* := \{ x \in X : \theta(x) > 0 \} \)
HJB Equations II: Customers

- Customer of firm \((n, x)\), given \(\omega = \{p, x'_{n-1}, x'_{n+1}\}\):

\[
rv^B(n, \omega) = v - p + (\delta_f + \delta_c) \left( U^B - V^B(n, \omega) \right) + (n - 1)\delta_c \left( x'_{n-1} - V^B(n, \omega) \right) + \eta \left( \theta(x'_{n+1}) \right) \left( x'_{n+1} - V^B(n, \omega) \right)
\]

- Flow surplus
- Separation
- Seller shrinks
- Seller grows

- Customer internalizes future prices through \(\theta : \mathcal{X} \to \mathbb{R}_+\) equilibrium mapping.
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rV^B(n, \omega) = v - p + (\delta_f + \delta_c)\left(U^B - V^B(n, \omega)\right)
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- Flow surplus
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- \(+(n-1)\delta_c\left(x'_{n-1} - V^B(n, \omega)\right)\)
  - Seller shrinks
- \(+\eta\left(\theta(x'_{n+1})\right)\left(x'_{n+1} - V^B(n, \omega)\right)\)
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HJB Equations III: Sellers

- **Incumbent firm** \((n, x) \rightarrow \) Contract \(\omega = \{p, x'_{n-1}, x'_{n+1}\} : \)

\[
rV^S(n, x) = \max_{\omega} \left\{ pn - C(n) + \delta_f \left( V^S_0 - V^S(n, x) \right) \right. \\
\left. + \delta_c \left( V^S(n-1, x'_{n-1}) - V^S(n, x) \right) + \eta \left( \theta(x'_{n+1}) \right) \left( V^S(n+1, x'_{n+1}) - V^S(n, x) \right) \right\} \\
\]

subject to promise-keeping:

\[
V^B(n, \omega) \geq x \quad \text{[PK]} 
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+ n\delta_c \left( V^S(n-1, x'_{n-1}) - V^S(n, x) \right) \\ \text{Death shock} \\
+ \eta \left( \theta(x'_{n+1}) \right) \left( V^S(n+1, x'_{n+1}) - V^S(n, x) \right) \\ \text{Gain a customer} \end{array} \right. \]

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- **Entrants** \((n = 0) \rightarrow\) Contract \(\omega = \{x'_1\} . \) By free entry: \(\kappa = \eta(\theta(x'_1)) V^S(1, x'_1)\)
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 \text{Lose a customer} \\
 + \kappa \left( \theta(x'_{n+1}) \right) \left( V^S(n+1, x'_{n+1}) - V^S(n, x) \right) \\
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\]
Recap:

- Transitions:

\[
\theta(x')_{n+1} \text{ s.t. free entry condition holds.}
\]

\[
\eta(\theta(x'_{n+1})) = \kappa V_S(1, x'_{n+1})
\]

\[
\begin{align*}
\delta f & + \delta c \eta(\theta(x'_{n+1})) = \kappa V_S(1, x'_{n+1}) \\
& \quad \text{Flow Equations}
\end{align*}
\]
Recap:

- Transitions: (i) grow by one;
Seller Transitions

Recap:

- Transitions: (i) grow by one; (ii) shrink by one;

\[
\eta(\theta(x_1')) \quad \eta(\theta(x_n')) \quad \eta(\theta(x_{n+1}'))
\]

\[
\delta_c \quad n\delta_c \quad (n+1)\delta_c
\]
Recap:

- Transitions: (i) grow by one; (ii) shrink by one; (iii) lose all;
Recap:

- Transitions: (i) grow by one; (ii) shrink by one; (iii) lose all;
- \(\{x_n'\}\)'s make buyers ex-ante indifferent, with \(x_1'\) s.t. free entry condition holds.

Flow Equations
Joint Surplus Problem

- In equilibrium, promise-keeping binds \( \Rightarrow V^B(n, \omega) = x \)

- Seller’s problem is equivalent to Joint Surplus problem:

\[
W(n, x) := V^S(n, x) + nx
\]

- Joint surplus

\[
\text{Seller} + nx
\]

\[
\text{Customers}
\]

\[
\text{Lose a customer}
\]

\[
\text{Gain a customer}
\]

\[
W(n, x) \text{ is invariant to (i) price } p; \text{ (ii) promised utility } x \Rightarrow \text{ Write: } W(n) = W(n, x)
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\[
(r + \delta_r) W(n, x) = \max_{x_{n+1} \in \Gamma_{n+1}} \left\{ n \left[ v + (\delta_r + \delta_c) U^B \right] - C(n) + \eta(\theta(x'_{n+1})) x'_{n+1} \right\}
\]

Customers’ flow surplus

- Customers’ flow surplus
- Seller’s flow costs
- Lose a customer
- Gain a customer

\[
= C(n) + \eta(\theta(x'_{n+1})) \left( W(n+1, x'_{n+1}) - W(n, x) \right)
\]

\[
+ n\delta_c \left( W(n-1, x'_{n-1}) - W(n, x) \right)
\]

- Write: \( W(n, x) = W(n, x) \)
Joint Surplus Problem

- In equilibrium, promise-keeping binds \( V^B(n, \omega) = x \)

- Seller’s problem is equivalent to **Joint Surplus problem**: Formal Statement and Proof

  - **Joint surplus** \( \rightarrow W(n, x) := V^S(n, x) + nx \)

  \[
  (r + \delta_f)W(n, x) = \max_{x'_{n+1}, x'_{n-1}} \left\{ n \left[ v + (\delta_f + \delta_c)U^B \right] - \left[ C(n) + \eta(\theta(x'_{n+1}))x'_{n+1} \right] \right\} \\
  + n\delta_c \left( W(n-1, x'_{n-1}) - W(n, x) \right) \\
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- \( W(n, x) \) is invariant to (i) price \( p \); (ii) promised utility \( x \)
Joint Surplus Problem

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$$
(r + \delta_f)W(n, x) = \max_{x'_{n+1}, x'_{n-1}} \left\{ \begin{array}{l}
n\left[v + (\delta_f + \delta_c)U^B\right] - \left[C(n) + \eta\left(\theta(x'_{n+1})\right)x'_{n+1}\right] \\
\text{Customers' flow surplus} \\
\text{Seller's flow costs} \\
\text{Lose a customer} \\
\text{Gain a customer}
\end{array} \right. \\
+ n\delta_c \left( W(n - 1, x'_{n-1}) - W(n, x) \right)
$$

- $W(n, x)$ is invariant to (i) price $p$; (ii) promised utility $x$. 

Joint Surplus Problem

- In equilibrium, promise-keeping binds → $V^B(n, \omega) = x$

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  (r + \delta_f)W(n, x) = \max_{x'_{n+1}, x'_{n-1}} \left\{ \begin{align*}
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  &\quad + n\delta_c \left( W(n - 1, x'_{n-1}) - W(n, x) \right) \\
  &\quad + \eta(\theta(x'_{n+1})) \left( W(n + 1, x'_{n+1}) - W(n, x) \right) \end{align*} \right\}
  \]

  - Customers’ flow surplus
  - Seller’s flow costs
  - Lose a customer
  - Gain a customer

- \( W(n, x) \) is invariant to (i) price \( p \); (ii) promised utility \( x \) \( \Rightarrow \) Write: \( W_n = W(n, x) \)
Joint Surplus Problem

- In equilibrium, promise-keeping binds \( \rightarrow V^B(n, \omega) = x \)

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\[
W(n, x) := \underbrace{V^S(n, x)}_{\text{Seller}} + \underbrace{nx}_{\text{Customers}}
\]

\[
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\left. + \left[ \left( W(n-1, x'_{n-1}) - W(n, x) \right) + n\delta_c \left( W(n-1, x'_{n-1}) - W(n, x) \right) \right] \right. \\
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\]

- \( W(n, x) \) is invariant to (i) price \( p \); (ii) promised utility \( x \) \( \Rightarrow \) Write: \( W_n = W(n, x) \)
Joint Surplus Problem

- In equilibrium, PK constraint binds \( V^B(n, \omega) = x \)

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  Define joint surplus \( W(n, x) := V^S(n, x) + nx \)

\[
(r + \delta_f)W_n = \max_{x'_{n+1}} \left\{ \begin{array}{l}
    n \left[ v + (\delta_f + \delta_c)U^B \right] - \left[ C(n) + \eta(\theta(x'_{n+1}))x'_{n+1} \right] \\
    \text{Buyers' total surplus} & \text{Seller's total costs} \\
    + n\delta_c \left( W_{n-1} - W_n \right) \\
    \text{Lose a customer} \\
    + \eta(\theta(x'_{n+1})) \left( W_{n+1} - W_n \right) \\
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Solve in 2 stages: 
[1st] Choose \( x'_{n+1} \); [2nd] Given \( x'_{n+1} \), choose \( p_n \);
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  Buyers’ total surplus

  
  $$(r + \delta_f)W_n = \max_{x'_{n+1}} \left\{ n \left[ v + (\delta_f + \delta_c)U^B \right] - \left[ C(n) + \eta(\theta(x'_{n+1}))x'_{n+1} \right] \right\}$$:

  Sellers’ total costs

  
  $$(r + \delta_f)W_n = \max_{x'_{n+1}} \left\{ n \left[ v + (\delta_f + \delta_c)U^B \right] - \left[ C(n) + \eta(\theta(x'_{n+1}))x'_{n+1} \right] \right\}$$

  Lose a customer

  
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- $W(n, x)$ is invariant to (i) price $p$; (ii) promised utility $x$  ⇒ Write: $W_n = W(n, x)$
- Solve in 2 stages: [1st] Choose $x'_{n+1}$; [2nd] Given $x'_{n+1}$, choose $p_n$;
Stage 1: Promised Utility

Stage 1. Choose $x'_{n+1}$:

$$\frac{\partial \eta(\theta(x))}{\partial x} \bigg|_{x=x'_{n+1}} \left( W_{n+1} - W_n \right) = \frac{\partial \eta(\theta(x))}{\partial x} \bigg|_{x=x'_{n+1}} x'_{n+1} + \eta(\theta(x'_{n+1}))$$

- Expected gain from ↑ in $W$
- Cost of paying $x'_{n+1}$ dollars
- ↓ in price by promise-keeping

[FOC]
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\]

**Result**

*If marginal gain $[W_{n+1} - W_n]$ is ↘ in $n$, then optimal $x'_{n+1}$ is ↘ in $n.*
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[FOC]

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If marginal gain $[W_{n+1} - W_n]$ is ↘ in $n$, then optimal $x'_{n+1}$ is ↘ in $n$.

Figure: Numerical example with constant marginal costs (i.e. $C(n) \propto n$).
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If marginal gain $[W_{n+1} - W_n]$ is $\searrow$ in $n$, then optimal $x'_{n+1}$ is $\searrow$ in $n$.

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Result

*If marginal gain $[W_{n+1} - W_n]$ is $\downarrow$ in $n$, then optimal $x'_{n+1}$ is $\downarrow$ in $n$.**

- **Mechanism:** Rents *today* vs. *tomorrow*

  - Small seller $\rightarrow$ Base yet small $\rightarrow$ Prefers to grow
    Cannot extract much today
  
  - As seller grows $\rightarrow$ Increasingly prefers to extract $\rightarrow$ Growth slows down from growing base
Stage 1: Promised Utility

- **Example:** \( \eta(\theta) = \theta^\gamma, \gamma \in (0, 1). \)

**Ex-post gains:**

1. Customer \( \rightarrow \) \( x_{n+1} - U^B = \gamma (W_{n+1} - W_n - U^B). \)

2. Seller \( \rightarrow \) \( V^S_{n+1} - V^S_n = (1 - \gamma) (W_{n+1} - W_n - U^B) + n(x_n - x_{n+1}). \)

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W_{n+1} - W_n - U^B = \left( \frac{\Gamma^B}{\gamma} \right)^\gamma \left( \frac{\Gamma^S_n}{1 - \gamma} \right)^{1-\gamma}
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Customer gains

Seller gains

Decomposition (seller)
Stage 1: Promised Utility

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**Figure:**
- **Customer gains**
  - Ex-ante vs. Ex-post

- **Seller gains**
  - Ex-ante vs. Ex-post

- **Decomposition (seller)**
  - \( \Delta V^S_{n+1} \)
  - [A] vs. [B]
Stage 2: Price Level

Stage 2. Given \( \{x_i\} \)'s from Stage 1, use:

\[
x_n = V^B(n, \{p_n, x_{n-1}, x_{n+1}\})\quad \Rightarrow \quad \text{Back out } p_n
\]

Recall:

- **Small seller:** High \( x \) ⇒ Fast growth ⇒ As \( n \uparrow \), promise less (\( x_n \downarrow \) with \( n \)).
- Implementation ⇒ \( p_n \uparrow \) with \( n \).

Price Decomposition

More on \( p \) vs. \( x \)
Stage 2: Price Level

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**Figure:** Numerical example with constant marginal costs (i.e. \( C(n) \propto n \)).
Distribution Dynamics and Efficiency

- Let $S_n \equiv$ Measure of sellers with $n = 0, 1, 2, \ldots$ customers.

- **Flow Equations:**

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- There exists a unique invariant distribution.

Proposition

The Markov-Perfect Equilibrium is constrained-efficient.

This suggests:

- Revenue dispersion in data might not be all due to inefficient resource misallocation.
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Outline

1 Baseline Model

2 Data and Calibration

3 Quantitative Results
   1 Response to Aggregate Shocks
   2 Secular Trends in Markups
**Quantitative Model:** Baseline Model + Shocks

- **Fully anticipated exogenous shocks** → Continuous-time Markov chains:
  1. Aggregate: $\varphi \in \{\varphi, \ldots, \overline{\varphi}\}$;
  2. Idiosyncratic: $z \in \{\underline{z}, \ldots, \overline{z}\}$; Entrants draw some $z_e \sim \pi_z$.

- **Fundamentals:**
  - Seller’s technology: $C(n, z; \varphi) \leftarrow [Supply shocks]$.
  - Buyer’s technology: $v(\varphi), c(\varphi) \leftarrow [Demand shocks]$.

- **Model Overview:**
  1. Pricing → Commitment + No Discrimination
  2. Sol’n method → Maximize joint surplus.
  3. Tractability → Equilibrium is block-recursive.

- All insights go through, including Analytical Characterization and Efficiency.
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All insights go through, including Analytical Characterization and Efficiency.
Quantitative Model: Baseline Model + Shocks

- Fully anticipated exogenous shocks → Continuous-time Markov chains:
  1. Aggregate: \( \phi \in \{ \varphi, \ldots, \bar{\varphi} \} \);
  2. Idiosyncratic: \( z \in \{ z, \ldots, \bar{z} \} \); Entrants draw some \( z_e \sim \pi_z \).

- Fundamentals:
  - Seller’s technology: \( C(n, z; \varphi) \) ← [Supply shocks].
  - Buyer’s technology: \( v(\varphi), c(\varphi) \) ← [Demand shocks].

- Model Overview:
  1. Pricing ⇒ Commitment + No Discrimination
  2. Sol’n method ⇒ Maximize joint surplus.
  3. Tractability ⇒ Equilibrium is block-recursive.

- All insights go through, including Analytical Characterization and Efficiency.
Calibration

**Parametrization:**

- **Costs:**  \( C(n; z, \varphi) = w e^{z+\varphi} n^\psi \)  
  \( \psi \geq 1; \ w > 0 \)

- **Shocks:**  \( d \log s_t = -\rho_s \log s_t \, dt + \sigma_s \, dB_t \)  
  \( s_t \in \{z_t, \varphi_t\}; \ B_t \sim BM \)

- **Matching:**  \( \eta(\theta) = \theta^\gamma \)  
  \( \gamma \in (0, 1) \)

**Strategy:**

- Calibrate by SMM to micro-pricing data from the U.S. retail sector.
- IRI Symphony scanner database → Week × Market × Store × Product level.
- Sample: 2001-2007; 59M transactions; 3.6k prod/store.
- Use relative prices:

  \[ \hat{p}_{us,t} = \log P_{us,t} - \frac{1}{N_{ut}^S} \sum_{s=1}^{N_{ut}^S} \log P_{us,t} \]

  Legend:  \( u = \) product;  \( s = \) store;  \( t = \) week;  \( N_{ut}^S = \# \{ \text{stores selling } u \text{ in week } t \} \)

- Key targets: (i) correlation growth-price; (ii) average markup.
Calibration

- **Parametrization:**

  \[ C(n; z, \varphi) = w e^{z+\varphi} n^\psi \quad \psi \geq 1; \ w > 0 \]

  \[ \text{Shocks : } \ d \log s_t = -\rho_s \log s_t \, dt + \sigma_s \, dB_t \quad s_t \in \{ z_t, \varphi_t \}; \ B_t \sim \text{BM} \]

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  - **Key targets:** (i) correlation growth-price; (ii) average markup.
# Model Fit

## Baseline Calibration: Model versus Data

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
<th>Target</th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu )</td>
<td>Value of consumption</td>
<td>Normalization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r )</td>
<td>Discount rate</td>
<td>5% risk-free rate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \delta_c )</td>
<td>Separation rate</td>
<td>.44% weekly rate</td>
<td></td>
<td></td>
<td>Paciello, Pozzi, Trachter (‘17)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Matching elasticity</td>
<td>Average markup</td>
<td>1.388</td>
<td>1.383</td>
<td>U.S. Census</td>
</tr>
<tr>
<td>( \delta_f )</td>
<td>Seller death rate</td>
<td>Entry rate</td>
<td>0.087</td>
<td>0.089</td>
<td>IRI</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Cost curvature param.</td>
<td>Price dispersion</td>
<td>0.1072</td>
<td>0.1055</td>
<td>IRI</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Cost scale param.</td>
<td>p50-p10 IDR in ( \hat{p} )</td>
<td>1.1224</td>
<td>1.1215</td>
<td>IRI</td>
</tr>
<tr>
<td>( c )</td>
<td>Buyer search cost</td>
<td>Average store size</td>
<td>10.73</td>
<td>12.44</td>
<td>IRI</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Firm entry cost</td>
<td>( \text{CORR}(\hat{p}, \Delta Sales) )</td>
<td>−0.023</td>
<td>−0.007</td>
<td>IRI</td>
</tr>
<tr>
<td>( \rho_z )</td>
<td>Persistence of ( z )</td>
<td>( \text{AC}(Sales) )</td>
<td>0.854</td>
<td>0.828</td>
<td>IRI</td>
</tr>
<tr>
<td>( \sigma_z )</td>
<td>Volatility of ( z )</td>
<td>( \text{SD}(Sales) )</td>
<td>0.6</td>
<td>0.474</td>
<td>IRI</td>
</tr>
</tbody>
</table>

**Freq.** Yearly

**Table:** Baseline calibration: model versus data.

**Notes:** Price dispersion computed as standard deviation of relative prices across all dates and products. Average store size computed as the within-store average number of physical units sold. Entry rate computed as the average across years of the ratio of the number of new stores (less than 1Y old) to all stores. Sales measures in the data are normalized by their mean.
Value and Policy Functions

\[ W_n(z) \]

(a) Joint Surplus

\[ p_n(z) \]

(b) Pricing Policy Function

\[ m_n(z) \times 10^{-3} \]

(c) Sales-weighted Markups

\[ z_n(\alpha) \]

(d) Promised Utility

**Figure:** Stationary value and policy functions for the calibrated set of parameters.
Validation

Non-targeted moments:

1. Upper- and lower-tail measures of relative price dispersion.
2. Moments of the distribution of price changes (IRI sample).

<table>
<thead>
<tr>
<th>Moment Model Data</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Distribution of Relative Prices</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p90-p10 range</td>
<td>1.1994</td>
<td>1.2504</td>
</tr>
<tr>
<td>p90-p50 range</td>
<td>1.0508</td>
<td>1.1149</td>
</tr>
<tr>
<td><strong>B. Distribution of Price Changes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average frequency</td>
<td>0.9639</td>
<td>0.9609</td>
</tr>
<tr>
<td>Median frequency</td>
<td>0.9814</td>
<td>0.9264</td>
</tr>
<tr>
<td>Average absolute price change</td>
<td>0.0305</td>
<td>0.0313</td>
</tr>
<tr>
<td>Median absolute price change</td>
<td>0.0312</td>
<td>0.0197</td>
</tr>
<tr>
<td>Absolute price change dispersion</td>
<td>0.0505</td>
<td>0.1415</td>
</tr>
</tbody>
</table>

Table: Non-targeted moments: model vs. data. Notes: Data from IRI Symphony, 2001-2007.
Outline

1. Baseline Model
2. Data and Calibration
3. Quantitative Results
   1. Response to Aggregate Shocks
   2. Secular Trends in Markups
**Figure:** Responses to an aggregate and transitory 1% increase in marginal costs.

Notes: diagrams plot cross-sectional averages using the theoretical distribution of sellers over states, $g_n(z) = S_n(z)/\Sigma_n z S_n(z)$. 

Variables Formulas

Price Margins

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Diagrams plot cross-sectional averages using the theoretical distribution of sellers over states, $g_n(z) = S_n(z)/\Sigma_n z S_n(z)$.
Effects on $x$ are *dampened* if relationships are *shorter* (i.e. $\delta_c$ higher). Compare:

1. Calibrated $\delta_c$ [ — ] \( \Rightarrow \) Avg duration \( \approx \) 4.9 years;
2. Higher $\delta_c$ [ --- ] \( \Rightarrow \) Avg duration \( \approx \) 3.3 years (*one-third* shorter);
Duration of Customer Relationships

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![Graphs showing buyer value, seller value, price, and measured markup over years for different $\delta_c$.]

**Figure:** Responses to 1% aggregate shock to marginal cost, for different values of $\delta_c$.

*Notes:* diagrams plot cross-sectional averages using the theoretical distribution of sellers over states, $g_n(z) = s_n(z)/\Sigma_{n,z} s_n(z)$. 

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Figure: Responses to an aggregate and transitory 1% decline in utility \( (v) \).

Notes: diagrams plot cross-sectional averages using the theoretical distribution of sellers over states, \( g_n(z) = \frac{s_n(z)}{\Sigma_{n,z}s_n(z)} \).
Secular Trends: U.S. Data

Since early 1980s:

1. A steady decline in business dynamism (entry rate).
2. A secular increase in the average markup.

**Figure:** Left: Change in firm entry rate since 1980. Source: U.S. Census Bureau, BDS. Right: Change in sales-weighted average markup since 1980. Source: De Loecker and Eeckhout (2017)
Secular Trends: Comparing Steady States

- **Exercise:** Decline in search costs (new shopping technologies, Internet, etc.).
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1. Prices ↑ ⇒ Why? $x_{n+1}$ must ↑ to keep customers locked in, so $p_n$ ↑
2. Entry ↓ ⇒ Why? Less buyers left unmatched, scope for entry ↓

Figure: Change in entry rate and average markup (stationary solution), for different search costs ($c$). Values normalized to 1 when $c = 1.2$. Vertical red dashed marks calibrated value.
Secular Trends: Comparing Steady States

**Exercise:** Decline in search costs (new shopping technologies, Internet, etc.).

1. Prices $\uparrow \Rightarrow$ **Why?** $x_{n+1}$ must $\uparrow$ to keep customers locked in, so $p_n \uparrow$

2. Entry $\downarrow \Rightarrow$ **Why?** Less buyers left unmatched, scope for entry $\downarrow$

3. Right-shift $\Rightarrow$ **Why?** Firm growth accelerates.

**Figure:** Change in entry rate and average markup (stationary solution), for different search costs ($c$). Values normalized to 1 when $c = 1.2$. Vertical red dashed marks calibrated value.
Secular Trends: Transitional Dynamics

- In the transition:
  1. Rise in cross-sectional markup dispersion.
  2. Trend driven by firms at the top of markup distribution.

**Figure:** Evolution of markup distribution after gradual decrease in search costs (in changes w.r.t. 1980). Simulation-based results (1k firms, 1k years, period length $dt = 0.01$).
Conclusion

- We study the implications of pricing when firms need to attract customers.

- **Micro implications:**
  - Smaller sellers set lower prices.
  - Take time to grow but experience faster growth rates.

- **Macro implications:**
  1. Price pass-through to aggregate shocks is incomplete, more so for smaller sellers.
  2. Average markup procyclical to supply and demand shocks.

Thank you!
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Thank you!
Appendix

1 Literature.
2 Proof of Equivalence.
3 $\theta(x)$ Diagram.
4 Other Size Relationship.
5 Cobb-Douglas Example.
6 Price Decomposition.
7 Inter-Temporal Trade-Off.
8 Invariant Distribution ($\delta_f = 0$).
9 Planner’s Problem.
10 HJB Equations General Model.
11 Extensions.
12 Distribution Dynamics.
13 Numerical Implementation.
14 Data Appendices.
15 $c$ Shock.
Plant-level data:
- Physically homogenous goods from the Census.
- This is to avoid quality differences → Prices reflect horizontal differentiation.

2 measures of TFP (plant \(i\), time \(t\)) → Output net of inputs (elasticities imputed by costs):

\[
\ln TFP_{it} = \ln q_{it} - \alpha_\ell \ln \ell_{it} - \alpha_k \ln k_{it} - \alpha_m \ln m_{it} - \alpha_e \ln e_{it}
\]

with:
1. \(TFPQ\) → Physical productivity → Dispersion in physical efficiency.
2. \(TFPR\) → Revenue productivity → Dispersion in profitability.

Procedure:
1. Regress:

\[
\ln q_{it} = \alpha_0 + \alpha_1 \ln p_{it} + YEAR_t + \eta_{it}
\]

where \(p_{it}\) instrumented by \(TFPQ\).

2. Retrieve idiosyncratic demand component of plant \(i\) as residual of IV regression.

Findings: 1. Idiosyncratic demand is (i) highly persistent; (ii) key determinant of survival.
2. Smaller plants set lower prices.
Estimate elasticities of substitution across varieties between and within firm.

Decompose firm-size distribution into contributions by:

a. *Marginal costs* → Affect revenue through prices.

b. “Appeal” (i.e. *quality or taste*) → Affect revenue *conditional on* prices.

c. *Markups* → Affect revenue through firm’s market power.

d. *Product scope* (# products) → Affect revenue through *cannibalization* effects.

**Finding:** Most differences in revenue productivity are *demand-driven* (i.e. appeal and scope).

**Cross-section** (% of variance in firm size):

1. **50-70%** → Due to differences in *firm appeal*.

2. **20-25%** → Due to differences in *product scope*.

3. **<25%** → Due to differences in *marginal costs*.

**Time series:**

1. Virtually *all* firm growth attributed to firm appeal.

2. Most of the remainder due to product scope.
Use *production approach* to compute markups → No need to estimate demand systems.

1. Firm $i$'s production function: $Q_{it}(\cdot) = F(V_{it}, K_{it})$. Cost minimization problem:

\[
\min_{V_{it}, K_{it}} P_{it}^V V_{it} + r_{it} K_{it} - \Lambda_{it} Q_{it}(\cdot), \quad \Lambda \equiv \text{Lagrange multip.} \quad \rightarrow \quad \text{FOC}: \quad \frac{P_{it}^V V_{it}}{Q_{it}} = \Lambda_{it} \varepsilon_Q^V
\]

where $\varepsilon_V^Q \equiv \text{Output elasticity of input}$.

2. $\Lambda$ proxies marginal cost, so $Mkp_{it} \equiv \frac{P_{it}^Q}{\Lambda_{it}}$. Recover $Mkp_{it}$ using FOC:

\[
Mkp_{it} = \varepsilon_V^Q \cdot \frac{P_{it}^Q Q_{it}}{P_{it}^V V_{it}} \quad \Rightarrow \quad \text{where} \quad \begin{cases} \frac{P_{it}^Q Q_{it}}{P_{it}^V V_{it}} & \text{from data} \\ \varepsilon_V^Q & \text{need to estimate} \end{cases}
\]

3. Posit production function (in logs) → $q_{it}(\cdot) = \beta_v V_{it} + \beta_k K_{it} + \omega_{it} + \varepsilon_{it}$.  
   \[
   \begin{array}{cccc}
   \text{Var. input} & \beta_v V_{it} \\
   \text{Capital} & \beta_k K_{it} \\
   \text{Productivity} & \omega_{it} \\
   \text{Meas. Err.} & \varepsilon_{it} \\
   \end{array}
   \]

4. Estimate $\beta_v$ using panel of firms à la Olley-Pakes (1996). Then → $\mu_{it} = \beta_v \frac{Sales_{it}}{\text{VarCost}_{it}}$.

**Findings:**

1. $\text{CORR}(\text{markup, firm size}) > 0$ within narrowly-defined industries.

2. Markup ↑ 3-fold, much action in upper tail of distribution.
Proposition (Equivalence)

1. Given a $\omega = \{p, x'(n')\}$ that solves seller’s problem, $x'(n')$ maximizes $W$.

2. For any $x'(n')$ that maximizes $W$, $\exists p$ s.t. $\{p, x'(n')\}$ solves the seller’s problem.

**Proof:**

- Let $\omega_n = \{p_n, x'_{n-1}, x'_n\}$ denote a policy. Let $\rho_n(x'_{n+1}) \equiv r + \delta_f + n\delta_c + \eta(\theta(x'_{n+1}))$.
- We can write seller’s problem as:
  \[
  V^S(n, x) = \max_{\omega_n} \tilde{V}^S(n; \omega_n) \quad \text{subject to} \quad x \leq V^B(n, \omega_n)
  \]
  where
  \[
  \tilde{V}^S(n; \omega_n) \equiv \frac{1}{\rho_n(x'_n)} \left[ \rho_n - C(n) + \eta(\theta(x'_n)) V^S(n + 1, x'_n) + n\delta_c V^S(n - 1, x'_{n-1}) \right]
  \]
- From [PK] with equality, back out price level under policy $\omega_n$ (call it $p^{PK}$):
  \[
p^{PK}(x; \{x'_{n+1}, x'_{n-1}\}) = \nu - \rho_n(x'_{n+1})x + (\delta_c + \delta_f)U^B + \eta(\theta(x'_{n+1}))x'_n + \delta_c(n - 1)x'_{n-1}
  \]
Proof of Equivalence (2 of 2)

- Replacing \( p^{PK} \) into \( \tilde{V}^S \), we obtain:

\[
\tilde{W}(n; \bar{\omega}_n) = \frac{1}{\rho_n(x'_{n+1})} \left[ n \left( v + (\delta_c + \delta_f)U^B \right) - \left( C(n) + \eta(\theta(x'_{n+1})x'_{n+1}) \right) 
\right.
\]
\[
+ \eta(\theta(x'_{n+1})) W(n+1, x'_{n+1}) + n\delta_c W(n-1, x'_{n-1}) \right]
\]

where

\[
\tilde{W}(n; \bar{\omega}_n) \equiv \tilde{V}^S(n; \bar{\omega}_n) + nx \quad \text{and} \quad W(n, x) = \max_{\bar{\omega}_n} \tilde{W}(n; \bar{\omega}_n)
\]

- Note that RHS of \( \tilde{W}(n; \bar{\omega}_n) \) is independent of \( x \) and \( p \). Write as \( \tilde{W}_n(\{x'_{n-1}, x'_{n+1}\}) \).

- Thus, optimal contract \( \omega^*_n = \{p^*_n, x'_{n-1}, x'_{n+1}\} \) is such that:

\[
\{x'_{n-1}, x'_{n+1}\} = \arg \max \tilde{W}_n(\{x'_{n-1}, x'_{n+1}\}) \quad \text{and} \quad p^*_n = p^{PK}(x'_{n}; \{x'_{n+1}, x'_{n-1}\})
\]

[See paper for generalization to model with (z, \varphi) shocks]
Equilibrium Transitions

Figure: Equilibrium tightness function $\theta : \mathcal{X} \rightarrow \mathbb{R}_+$, and set of equilibrium markets when $W_n$ is concave.
Equilibrium Transitions

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**Equilibrium Transitions**

Figure: Equilibrium tightness function $\theta : \mathcal{X} \to \mathbb{R}_+$, and set of equilibrium markets when $W_n$ is concave.
Equilibrium Transitions

\[ \text{Promised utility (} x \text{)} = \text{Buyer-Seller ratio (} \theta \text{)} \cdot \theta(x) \cdot n \delta_c \cdot x_{n-1} \]

**Figure:** Equilibrium tightness function \( \theta : x \rightarrow \mathbb{R}_+ \), and set of equilibrium markets when \( W_n \) is concave.
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Figure: Equilibrium tightness function $\theta : \mathcal{X} \to \mathbb{R}_+$, and set of equilibrium markets when $W_n$ is concave.
If $\kappa$ is low enough, $p_n \downarrow$ in $n$ may emerge.

**Figure:** Numerical example with constant marginal costs (i.e. $C(n) \propto n$), and low value for $\kappa$. 
When $\eta(\theta) = \theta^\gamma$, $\gamma \in (0, 1)$, the [IC] condition implies $\theta(x) = \left(\frac{c + rU^B}{x - U^B}\right)^{\frac{\gamma}{\gamma - 1}}$.

With this, the [FOC] yields $x_{n+1} = \gamma(W_{n+1} - W_n) + (1 - \gamma)U^B$.

Plugging into [HJB] for $W_n$, we obtain a 2nd Order Difference Equation:

$$W_{n+1} = W_n + U^B + \left(\frac{\Gamma^B}{\gamma}\right)^\gamma \left(\frac{\Gamma^S_n}{1 - \gamma}\right)^{1-\gamma}$$

where $W_0 = 0$, and

$$\Gamma^B \equiv c + rU^B$$

$$\Gamma^S_n \equiv \begin{cases} \kappa & \text{if } n = 0 \\ (r + \delta_f)W_n - n\left(\nu - C(n)/n + (\delta_c + \delta_f)U^B\right) + n\delta_c(W_n - W_{n-1}) & \text{if } n \geq 1 \end{cases}$$

Now, note:

By [IC] : $\Gamma^B = \mu(\theta_{n+1})(x_{n+1} - U^B) \quad \Rightarrow \quad Buyer's \ ex-ante \ value \ of \ matching$

By [HJB] : $\Gamma^S_n = \eta(\theta_{n+1})(W_{n+1} - W_n - x_{n+1}) \quad \Rightarrow \quad Seller's \ ex-ante \ value \ of \ matching$

Thus $\Rightarrow$ Gain in JS is a convex combination between the two ex-ante match surpluses.
From $x_n = V^B(n; \{p_n, x_{n-1}, x_{n+1}\})$, price level can be decomposed as follows:

$$p_n = \underbrace{v - rx_n}_{\in [0,v] \text{ “Baseline” margin}} + \underbrace{\delta_f(U^B - x_n)}_{\leq 0 \text{ “Exit” margin}} + \underbrace{\eta(\theta_{n+1})(x_{n+1} - x_n)}_{\leq 0 \text{ “Growth” margin}} + \underbrace{n\delta_c \left( \frac{U^B + (n - 1)x_{n-1}}{n} - x_n \right)}_{\leq 0 \text{ “Separation” margin}}$$
From $x_n = V^B(n; \{p_n, x_{n-1}, x_{n+1}\})$, price level can be decomposed as follows:

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$$p_n = v - rx_n + \delta_f (U^B - x_n) + \eta(\theta_{n+1})(x_{n+1} - x_n) + n\delta_c \left( \frac{U^B + (n-1)x_{n-1}}{n} - x_n \right)$$

**Figure:** Numerical example with constant marginal costs (i.e. $C(n) \propto n$).
From $x_n = V^B(n; \{p_n, x_{n-1}, x_{n+1}\})$, price level can be decomposed as follows:

$$p_n = \begin{cases} v - rx_n & \text{Baseline margin} \\ \delta_f (U^B - x_n) & \leq 0 \text{ Exit margin} \\ \eta (\theta_{n+1}) (x_{n+1} - x_n) & \leq 0 \text{ Growth margin} \\ n\delta_c \left( \frac{U^B + (n-1)x_{n-1}}{n} - x_n \right) & \leq 0 \text{ Separation margin} \end{cases}$$

**Figure:** Numerical example with constant marginal costs (i.e. $C(n) \propto n$).
Sellers’ Inter-Temporal Trade-Off

Using $p_n$ (profit today) vs. $x'_{n+1}$ (profit tomorrow):

- Marginal $\$ \text{ promised...}$
Sellers’ Inter-Temporal Trade-Off

- Using $p_n$ (profit today) vs. $x'_{n+1}$ (profit tomorrow):

\[
\frac{\partial p_n}{\partial x'_{n+1}} = \eta (\theta_n + 1)
\]

Marginal $\$$ promised...

(A) ...is financed via $p_n \uparrow$, but...

(B) ...by commitment, $p_n \downarrow$ too, because:

(B.1) Extra $ raises probability of new buyer.

(B.2) Pre-existing customers require $p_n \downarrow$ as compensation.
Using $p_n$ (profit today) vs. $x_{n+1}'$ (profit tomorrow):

$$\frac{\partial p_n}{\partial x_{n+1}'} = \eta(\theta_{n+1}) + \underbrace{\eta(\theta_{n+1})}_{{(A)>0}}$$

Marginal $\$ \text{ promised...}$

(A) \text{...is financed via } p_n \uparrow, \text{ but...}$

(B) \text{...by commitment, } p_n \downarrow \text{ too, because:}$
Sellers’ Inter-Temporal Trade-Off

- Using $p_n$ (profit today) vs. $x'_{n+1}$ (profit tomorrow):

\[
\frac{\partial p_n}{\partial x'_{n+1}} = \eta(\theta_{n+1}) + \frac{\partial \eta(\theta_{n+1})}{\partial x'_{n+1}} \left( x'_{n+1} - x'_{n} \right)
\]

(A) $> 0$

(B.1) $> 0$

(B.2) $< 0$

Marginal $\$ promised...

(A) ...is financed via $p_n \uparrow$, but...

(B) ...by commitment, $p_n \downarrow$ too, because:

(B.1) Extra $\$ raises probability of new buyer.

(B.2) Pre-existing customers require $p_n \downarrow$ as compensation.
Using $p_n$ (profit today) vs. $x'_{n+1}$ (profit tomorrow):

$$\frac{\partial p_n}{\partial x'_{n+1}} = \eta(\theta_{n+1}) + \frac{\partial \eta(\theta_{n+1})}{\partial x'_{n+1}} \left( x'_{n+1} - x'_n \right) > 0$$

(A) $> 0$

Marginal $p_n$ promised...

(A) ...is financed via $p_n \uparrow$, but...

(B) ...by commitment, $p_n \downarrow$ too, because:

(B.1) Extra $ raises probability of new buyer.

(B.2) Pre-existing customers require $p_n \downarrow$ as compensation.

In equilibrium, $x'_{n+1} \uparrow$ implies $p_n \uparrow$
■ Using $p_n$ (profit today) vs. $x'_{n+1}$ (profit tomorrow):

\[
\frac{\partial p_n}{\partial x'_{n+1}} = \eta(\theta_{n+1}) + \frac{\partial \eta(\theta_{n+1})}{\partial x'_{n+1}} (x'_{n+1} - x'_{n}) > 0
\]

(A) $> 0$

(B.1) $> 0$

(B.2) $< 0$

(B) $< 0$

Marginal $\$ promised...

(A) ...is financed via $p_n \uparrow$, but...

(B) ...by commitment, $p_n \downarrow$ too, because:

(B.1) Extra $\$ raises probability of new buyer.

(B.2) Pre-existing customers require $p_n \downarrow$ as compensation.
Special Case: Analytical Solution

- **Case** $\delta_f = 0$:  \Rightarrow \text{Analytical solution} for stationary distribution:
  
  - Let $g_{n,t} \equiv S_{n,t}/S_t$ (share of incumbents of size $n$).
  
  - Stationary solution:

  \[
  \forall n \geq 1: \quad g_n = \frac{S_0}{S} \frac{\delta_c^{-n}}{n!} \prod_{i=0}^{n-1} \eta(\theta_{i+1}), \quad \text{with} \quad \frac{S_0}{S} = \left[ \sum_{n=1}^{+\infty} \frac{\delta_c^{-n}}{n!} \prod_{i=0}^{n-1} \eta(\theta_{i+1}) \right]^{-1},
  \]

  Solution exists so long as
  \[
  \sum_{n=1}^{+\infty} \frac{\delta_c^{-n}}{n!} \prod_{i=0}^{n-1} \eta(\theta_{i+1}) < +\infty.
  \]

- **Properties** of the solution:
  
  - When $x_n \downarrow U^B$ with $n$ (standard case)  \Rightarrow Size distribution has fat right-tail.
  
  - Tail’s fatness depends on rate of decay of $x_n$ with $n$.
  
  - Model with $\delta_f > 0$  \Rightarrow Thinner tail.
Planner’s Problem (1 of 2)

\[
\max_{\{B_{n,t}^A, B_{n,t}^I, S_{n,t} \}_{n \in \mathbb{N}}, \{\theta_{n,t} \}_{n \in \mathbb{N}}} \quad \mathbb{E}_0 \int_0^{+\infty} e^{-rt} \left\{ -\kappa S_{0,t} + \sum_{n=1}^{+\infty} \left( vB_{n,t}^A - C(n)S_{n,t} - cB_{n,t}^I \right) \right\} dt
\]

subject to:

1. **Distribution of sellers:**
   \[
   \partial_t S_{0,t} = \delta_f \sum_{n=1}^{+\infty} S_{n,t} + \delta_c S_{1,t} - \eta(\theta_{1,t})S_{0,t}
   \]
   \[
   \forall n \geq 1 : \quad \partial_t S_{n,t} = \eta(\theta_{n,t})S_{n-1,t} + (n + 1)\delta_c S_{n+1,t} - \left( \delta_f + n\delta_c + \eta(\theta_{n+1,t}) \right) S_{n,t}
   \]

2. **Distribution of buyers:**
   \[
   B_{n,t}^A = nS_{n,t}; \quad B_{n,t}^I = \theta_{n,t}S_{n-1,t}; \quad \sum_{n=1}^{+\infty} \left( B_{n,t}^A + B_{n,t}^I \right) = 1;
   \]

3. **Non-negative entry:** \( S_{0,t} \geq 0. \)
First, combine constraints on buyers’ distribution, and replace into objective function. Let \( \lambda_n \equiv \) multiplier on each \( \partial_t S_{n,t} \) constraint; \( \vartheta \equiv \) multiplier on \( \sum_{n=1}^{\infty} (B^A_{n,t} + B^I_{n,t}) = 1 \).

Optimality conditions (from Hamiltonian):

\[
\begin{align*}
    r\lambda_0 &= -\kappa - \theta_1(\vartheta + c) + \eta(\theta_1)(\lambda_1 - \lambda_0); \\
    \forall n \geq 1: \quad r\lambda_n &= n(n - \vartheta) - C(n) - (\vartheta + c)\theta_{n+1} + \delta_f(\lambda_0 - \lambda_n); \\
    &\quad + n\delta_c(\lambda_{n-1} - \lambda_n) + \eta(\theta_{n+1})(\lambda_{n+1} - \lambda_n) \\
    \forall n \geq 1: \quad \vartheta + c &= \frac{\partial\eta(\theta)}{\partial\theta} \bigg|_{\theta = \theta_n} (\lambda_n - \lambda_{n-1});
\end{align*}
\]

Finally, it is easy to check that planner’s and decentralized allocations coincide if:

\[
\begin{align*}
    \lambda_n &= W_n - nU^B \\
    \vartheta &= rU^B
\end{align*}
\]

[See paper for generalization to model with \((z,\varphi)\) shocks]}
HJB Equations I: Buyers

- Unmatched buyers \( \rightarrow U^B(\varphi) = \max_{x \in x} u^B(x; \varphi) \), where:

\[
r u^B(x; \varphi) = -c(\varphi) + \mu(\theta(x; \varphi)) \left( x - u^B(x; \varphi) \right) + \sum_{\varphi' \in \Phi} \lambda_{\varphi}(\varphi'|\varphi) \left( u^B(x; \varphi') - u^B(x; \varphi) \right)
\]

Search cost Option value of matching Aggregate shock

\( \Rightarrow \) Buyers' indifference: \( \forall \varphi : u^B(x; \varphi) \leq U^B(\varphi) \), with equality if \( \theta(x; \varphi) > 0 \) [IC].

- Customer of firm \((n, x; s)\), where \( s := (z, \varphi) \), given \( \omega = \{p, x'(n'; s')\} \):

\[
r V^B(n, \omega; s) = v(\varphi) - p + (\delta_f + \delta_c) \left( U^B(\varphi) - V^B(n, \omega; s) \right) + (n - 1)\delta_c \left( x'(n - 1; s) - V^B(n, \omega; s) \right) \\
+ \eta \left( \theta(x'(n + 1; s); \varphi) \right) \left( x'(n + 1; s) - V^B(n, \omega; s) \right) \\
+ \sum_{z' \in Z} \lambda_z(z'|z) \left( x'(n; z', \varphi) - V^B(n, \omega; s) \right) + \sum_{\varphi' \in \Phi} \lambda_{\varphi}(\varphi'|\varphi) \left( x'(n; z, \varphi') - V^B(n, \omega; s) \right)
\]

Idiosyncratic shock Aggregate shock
**HJB Equations II: Incumbent Sellers**

- **Incumbent firm of type** \((n, x; s)\):

  \[ rV^S(n, x; s) = \max_{\omega} \left\{ \begin{array}{l}
  pn - C(n; s) + \delta_f \left( V_0^S(\varphi) - V^S(n, x; s) \right) \\
  + n\delta_c \left( V^S(n - 1, x'(n - 1; s); s) - V^S(n, x; s) \right) \\
  + \eta \left( \theta(x'(n + 1; s); \varphi) \right) \left( V^S(n + 1, x'(n + 1; s); s) - V^S(n, x; s) \right) \\
  + \sum_{z' \in Z} \lambda_z(z'|z) \left( V^S(n, x'(n; z', \varphi); z', \varphi) - V^S(n, x; s) \right) \\
  + \sum_{\varphi' \in \Phi} \lambda_{\varphi}(\varphi'|\varphi) \left( V^S(n, x'(n; z, \varphi'); z, \varphi') - V^S(n, x; s) \right) \end{array} \} \]

  subject to:

  \[ V^B(n, \omega; s) \geq x \quad \text{[PK]} \]
Potential entrant (size $n = 0$):

- Chooses and commits to contract $\omega_0 = \{x'(1; z_0, \varphi)\}$, where $z_0 \sim \pi_z$.

\[
rv_0^S(\varphi) = -\kappa + \sum_{z_0 \in Z} \pi_z(z_0)v_0^S(z_0, \varphi) + \sum_{\varphi' \in \Phi} \lambda_{\varphi}(\varphi'|\varphi) \left( V_0^S(\varphi') - V_0^S(\varphi) \right)
\]

where

\[
v_0^S(z_0, \varphi) := \max_{x'(1; z_0, \varphi)} \eta \left( \theta \left( x'(1; z_0, \varphi); \varphi \right) \right) \left( V^S(1, x'(1; z_0, \varphi); z_0, \varphi) - V_0^S(\varphi) \right)
\]

Free entry $\rightarrow V_0^S(\varphi) = 0$, so:

\[
\kappa = \sum_{z_0 \in Z} \pi_z(z_0)v_0^S(z_0, \varphi), \quad \forall \varphi \in \Phi \quad \text{[FE]}
\]

Look for a BRE with positive entry in all aggregate states $\varphi$. 

Back to General Model
Extensions

1. Price Discrimination (PD):
   - The “No PD” assumption can be relaxed and still obtain firm dynamics.
   - Caveat $\rightarrow$ PD implies price indeterminacy (within the Markov Eq’m class).
   - Intuitively:

     $$\{p_i\}_{i=1}^{n} \text{ sustainable in eq’m } \iff \sum_{i=1}^{n} x_i \text{ maximizes } W_n \rightarrow \{x_i\} \text{ distribution is irrelevant!}$$

2. Commitment:
   - Reputational concern key to generate trade-offs. Without commitment:
     1. Multiplicity would emerge (e.g. Nakamura and Steinsson (2011)).
     2. Block-recursivity would be lost $\rightarrow$ Cannot solve with aggregate shocks.

3. Endogenous Customer Separations:
   - Customer’s search problem $\Rightarrow \Gamma(n, \omega) \equiv \max_{x \geq V^B(n, \omega)} \mu(\theta(x))(x - V^B(n, \omega))$
   - Model with shocks would need seller free entry into all markets (not just $n = 1$).
Extension: Price Discrimination (1 of 2)

- Contracts → \( \{ p_i, x_i'(n - 1), x_i'(n + 1) \}_{i=1}^n \), and \( x'_0 \) for incoming customer.
- Value of the seller:

\[
rV^S(n, \{x_i\}_{i=1}^n) = \max_{x'_0, \{\omega_i\}_{i=1}^n} \left\{ \sum_{i=1}^n p_i - C(n) + \delta_f \left( V^S_0 - V^S(n, \{x_i\}_{i=1}^n) \right) \right. \\
+ \delta_c \sum_{j=1}^n \left( V^S(n - 1, \{x_i'(n - 1)\}_{i=1}^n \setminus \{x_j'(n - 1)\}) - V^S(n, \{x_i\}_{i=1}^n) \right) \\
+ \eta(\theta(x'_0)) \left( V^S(n + 1, \{x_i'(n + 1)\}_{i=1}^n \cup \{x'_0\}) - V^S(n, \{x_i\}_{i=1}^n) \right) \right\} \\
\text{s.t. } x_i \leq V^B(n, \omega_i), \forall i = 1, \ldots, n.
\]

- We show [see paper] that problem is equivalent to maximizing:

\[
W(n, \{x_i\}_{i=1}^n) = V^S(n, \{x_i\}_{i=1}^n) + \sum_{i=1}^n x_i
\]

and that \( W \) is independent of the distribution of utilities ⇒ \( W(n, \{x_i\}_{i=1}^n) = W_n \).
Proposition (Indeterminacy)

There is a continuum of joint-surplus-maximizing contracts that leave both buyers and seller indifferent.

Proof [sketched]:

- Pick $\varepsilon > 0$ arbitrarily.
- To show:
  If $\omega^a = \{p_i, x'_i(n')\}_{i=1}^n$ is optimal, $\exists \beta_n$ s.t. $\omega^b = \{p_i + \varepsilon \beta_n, x'_i(n') + \varepsilon\}_{i=1}^n$ is optimal too.
- Some algebra shows that:

\[
V^B(n, \omega^b_i) = V^B(n, \omega^a_i) + \varepsilon \left( \beta_n - (n - 1)\delta_c - \eta(\theta(x'_0)) \right)
\]
\[
V^S(n, \{x_i\}_{i=1}^n; \omega^b_i) = V^S(n, \{x_i\}_{i=1}^n; \omega^a_i) + n\varepsilon \left( \beta_n - (n - 1)\delta_c - \eta(\theta(x'_0)) \right)
\]

- Thus, need $\beta_n = (n - 1)\delta_c + \eta(\theta(x'_0))$. $\square$

[See paper for generalization to model with $(z, \varphi)$ shocks]
Joint Surplus Problem

- The Seller’s and the Joint Surplus problems are equivalent, in the sense that:

$$(\Rightarrow) \quad \text{Given contract } \omega^*_n = \{ p_n, x'_n(n'; s') \} \text{ that solves Seller’s problem, } x'_n(n'; s') \text{ is a solution to:}$$

$$rW(n, x; s) = \max_{x'(n'; s')} \left\{ n \left( v + (\delta_f + \delta_c) U^B(\varphi) \right) - \left( C(n; s) + \eta \left( \theta \left( x'(n + 1; s); \varphi \right) \right) x'(n + 1; s) \right. \right.$$  

\[ \text{Customers’ total surplus} \]

\[ + n\delta_c \left( W\left( n - 1, x'(n - 1; s); s \right) - W(n, x; s) \right) - \delta_f W(n, x; s) \]

\[ + \eta \left( \theta \left( x'(n + 1; s); \varphi \right) \right) \left( W\left( n + 1, x'(n + 1; s); s \right) - W(n, x; s) \right) \]

\[ + \sum_{z' \in Z} \lambda_z(z'|z) \left( W\left( n, x(z', \varphi); z', \varphi \right) - W(n, x; s) \right) \]

\[ + \sum_{\varphi' \in \Phi} \lambda_{\varphi}(\varphi'|\varphi) \left( W\left( n, x(z, \varphi'); z, \varphi' \right) - W(n, x; s) \right) \left\} \right.$$  

\[ \text{Seller’s total costs} \]

$$(\Leftarrow) \quad \text{For } x'_n(n'; s') \text{ that solves JS problem, } \exists p_n \text{ s.t. } \{ p_n, x'_n(n'; s') \} \text{ solves Seller’s problem.}$$
A **block-recursive equilibrium** is, for all \((n, s) \in \mathbb{N} \times \mathbb{Z} \times \Phi\):

(i) **Value functions:**

\[
V^S(n, x; s), \ V^B(n, \omega; s), \ U^B(\varphi), \ V^S_0(\varphi), \ W(n, x; s);
\]

(ii) **Contracts:**

\[
\omega_n(s) = \{ p_n(s), x_n(s'), x_n^+(s), x_n^-(s) \}, \ \omega_0(\varphi) = \{ x_1(z_0, \varphi) \}_{z_0 \in \mathbb{Z}};
\]

(iii) **Market tightness function:**

\[
\theta(\cdot; \varphi) : \mathcal{X} \to \mathbb{R}_+;
\]

(iv) **Distributions:**

\[
\{ S_0(\varphi), S_n(z, \varphi); B^A_n(z, \varphi), B^I_n(z, \varphi) \};
\]

such that:

1. The value functions solve agents’ problems, with \(V_0(\varphi) = 0\) by free entry.
2. Contracts and promised utilities maximize the joint surplus.
3. Market tightness is consistent with the sorting behavior of inactive buyers.
4. Aggregates and the distribution of agents satisfy the KF equations.
**Block-Recursivity:**

- Dynamics across states uniquely pinned down by $\theta$’s.
- Key ingredients $\rightarrow$ (i) Directed search; (ii) Free entry of agents.

$\{\theta, x, p, W\}$ jump with $\varphi$, but measures of agents evolve smoothly.

¶ Exact characterization of dynamics in and out of steady state!
Distribution Dynamics (2 of 4)

- Buyers → \[
\begin{cases}
  \text{Active} : & B^A_n(z) = nS_n(z) \\  \text{Inactive} : & B^I_n(z, \varphi) = \theta_n(z, \varphi) \cdot S_{n-1}(z)
\end{cases} \quad [\text{by construction}]
\]

- Potential entrants (\( n = 0 \)):

\[
\partial_t S_{0,t}(\varphi) = \delta_f S_t + \delta_c \sum_{z \in \mathcal{Z}} S_{1,t}(z) - \sum_{z_0 \in \mathcal{Z}} \pi_z(z_0) \eta\left(\theta_{1,t}(z_0, \varphi)\right) S_{0,t}(\varphi)
\]

\( \underline{\text{Incumbents losing customers}} \quad \underline{\text{Actual entrants}} \)

**Notation:**

- \((z, \varphi)\) → (Idiosyncratic, Aggregate) state
- \(\theta_{n,t}(z, \varphi)\) → Tightness in mkt. for \(n^{th}\) customer \((\text{jump})\)
- \(S_{0,t}(\varphi)\) → Measure of potential entrants \((\text{jump})\)
- \(S_{n,t}(z)\) → Measure of firms size \(n\), productivity \(z\) \((\text{stock})\)
- \(S_t := \sum_{n \in \mathbb{N}} \sum_{z \in \mathcal{Z}} S_{n,t}(z)\) → Total mass of incumbent firms \((\text{stock})\)
Kolmogorov Forward Equations for seller type \((n, z) \in \mathbb{N} \times \mathcal{Z}\):

- **If** \(n = 1\):
  
  \[
  \frac{\partial}{\partial t} S_{1,t}(z) = \pi(z) \eta(t, z) S_{0,t}(\varphi) + 2\delta_c S_{2,t}(z) + \sum_{\tilde{z} \neq z} \lambda(z | \tilde{z}) S_{1,t}(\tilde{z}) \\
  - \left( \delta_f + \delta_c + \eta(t, z) + \sum_{\tilde{z} \neq z} \lambda(z | \tilde{z}) \right) S_{1,t}(z)
  \]

- **If** \(n \geq 2\):
  
  \[
  \frac{\partial}{\partial t} S_{n,t}(z) = \eta(t, z) S_{n-1}(z) + (n + 1)\delta_c S_{n+1,t}(z) + \sum_{\tilde{z} \neq z} \lambda(z | \tilde{z}) S_{n,t}(\tilde{z}) \\
  - \left( \delta_f + n\delta_c + \eta(t, z) + \sum_{\tilde{z} \neq z} \lambda(z | \tilde{z}) \right) S_{n,t}(z)
  \]
Kolmogorov Forward Equations for seller type \((n, z) \in \mathbb{N} \times \mathcal{Z}\):

- If \(n = 1\):
  \[
  \partial_t S_{1,t}(z) = \pi_z(z) \eta(\theta_1, t(z, \varphi)) S_{0,t}(\varphi) + 2\delta_c S_{2,t}(z) + \sum_{\tilde{z} \neq z} \lambda_z(z | \tilde{z}) S_{1,t}(\tilde{z}) - \left(\delta_f + \delta_c + \eta(\theta_2, t(z, \varphi)) + \sum_{\tilde{z} \neq z} \lambda_z(\tilde{z} | z)\right) S_{1,t}(z)
  \]

- If \(n \geq 2\):
  \[
  \partial_t S_{n,t}(z) = \underbrace{\eta(\theta_n, t(z, \varphi)) S_{n-1}(z)}_{\text{Gain } n^{th} \text{ customer}} + \underbrace{(n + 1)\delta_c S_{n+1,t}(z)}_{\text{Lose } (n+1)^{th} \text{ customer}} + \sum_{\tilde{z} \neq z} \lambda_z(z | \tilde{z}) S_{n,t}(\tilde{z}) - \left(\delta_f + n\delta_c + \eta(\theta_{n+1}, t(z, \varphi)) + \sum_{\tilde{z} \neq z} \lambda_z(\tilde{z} | z)\right) S_{n,t}(z)
  \]

- Outflows
Computing stationary aggregate measures of agents:

1. Let $S \equiv \sum_{n \geq 1} S_n$, $h_0 \equiv S_0 / S$, and $g_n \equiv S_n / S$, $n \geq 1$.

2. Then $b^A \equiv B^A / S = \sum_{n \geq 1} n g_n$.

3. Moreover, $B^l_n = S \theta_n g_{n-1}$. Thus:

$$S \sum_{n=2}^{+\infty} \theta_n g_{n-1} = \sum_{n=2}^{+\infty} B^l_n = B^l - B^l_1 = 1 - B^A - \theta_1 S_0 = 1 - (b^A + h_0 \theta_1) S$$

4. Solving for $S$:

$$S = \left( b^A + h_0 \theta_1 + \sum_{n=1}^{+\infty} \theta_{n+1} g_n \right)^{-1}$$

5. Once we know $S$, we can easily compute remaining aggregate measures of agents:

$$S_0 = S h_0 \quad B^A = S b^A \quad B^l = 1 - B^A$$
Numerical Implementation of State Processes

- In the model $s \in \{z, \varphi\} \sim$ Continuous-time Markov Chain with generator matrix:

$$\Lambda = \begin{pmatrix}
- \sum_{j \neq 1} \lambda_{1j} & \lambda_{12} & \cdots & \lambda_{1k_s} \\
\lambda_{21} & - \sum_{j \neq 2} \lambda_{2j} & \cdots & \lambda_{2k_s} \\
\vdots & \vdots & \ddots & \vdots \\
\lambda_{k_s1} & \lambda_{k_s2} & \cdots & - \sum_{j \neq k_s} \lambda_{k_s j}
\end{pmatrix}; \quad \lambda_{ij} \equiv s_i \to s_j \text{ rate.}
$$

- In practice $\Rightarrow \log s_t \sim$ Driftless Ornstein-Uhlenbeck process:

$$d \log s_t = -\rho_s \log s_t dt + \sigma_s dB_t$$

1. Discretize time to $T = \{\Delta, 2\Delta, 3\Delta, \ldots \}$. Use Euler-Maruyama method:

$$\forall k \in T: \quad \log s_k = (1 - \rho_s \Delta) \log s_{k-1} + \sigma_s \sqrt{\Delta} \varepsilon_k; \quad \varepsilon_k \sim iid \mathcal{N}(0, 1) \quad (1)$$

2. Estimate (1) by Tauchen method. Outcome $\Rightarrow$ Markov matrix $\Pi_s = (\pi_{ij})$, with $k_s = 25$.

3. Map back into continuous time using:

$$\forall i = 1, \ldots, k_s: \quad \pi_{ij} \approx \lambda_{ij} \Delta, \; \forall j \neq i \quad \text{and} \quad \pi_{ii} \approx 1 - \sum_{j \neq i} \lambda_{ij} \Delta$$
IRI Data: Sample Selection

- Weekly transacted prices (good $u$, store $s$, week $t$) $\Rightarrow P_{us,t} = \frac{TotalRevenue_{us,t}}{Units_{us,t}}$.

- Sample Selection:
  - Drop uncommon products (sold in less than 10 stores).
  - Focus on regular prices only (i.e. no promotions, no temporary sales).
  - Outliers: cap prices at $100$, drop non-positive sales and prices.

<table>
<thead>
<tr>
<th></th>
<th>Full Sample</th>
<th>Sub-Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td># chains</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td># stores</td>
<td>278</td>
<td>278</td>
</tr>
<tr>
<td># UPCs</td>
<td>19,721</td>
<td>11,483</td>
</tr>
<tr>
<td>Stores per chain</td>
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<td>25</td>
</tr>
<tr>
<td>Stores per product</td>
<td>59</td>
<td>88</td>
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<tr>
<td>Products per store</td>
<td>4,180</td>
<td>3,638</td>
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<tr>
<td>Average price</td>
<td>$7.75$</td>
<td>$8.32$</td>
</tr>
<tr>
<td>Price dispersion</td>
<td>15.73%</td>
<td>10.54%</td>
</tr>
<tr>
<td>Total Sales (in B)</td>
<td>$\approx$2.86</td>
<td>$\approx$1.60</td>
</tr>
<tr>
<td># weeks</td>
<td>365</td>
<td>365</td>
</tr>
<tr>
<td>MSAs considered</td>
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<td>NY, LA</td>
</tr>
<tr>
<td># transactions</td>
<td>89,112,170</td>
<td>59,813,217</td>
</tr>
</tbody>
</table>

Table: Descriptive statistics of the sample. Source: IRI Symphony weekly data.
Notes: Price dispersion computed as the average standard deviation of log-standardized prices.
A Look at the IRI Data

<table>
<thead>
<tr>
<th></th>
<th>Relative prices</th>
<th>Norm. sales</th>
<th>Sales growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0</td>
<td>1</td>
<td>-.0008</td>
</tr>
<tr>
<td>Median</td>
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<td>1st</td>
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<td>-.1905</td>
</tr>
<tr>
<td>10\textsuperscript{th}</td>
<td>-.1138</td>
<td>.4709</td>
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<td>25\textsuperscript{th}</td>
<td>-.0415</td>
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<td>-.0378</td>
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<td>75\textsuperscript{th}</td>
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<td>90\textsuperscript{th}</td>
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<td>1.7029</td>
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<td>99\textsuperscript{th}</td>
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<td>Dispersion</td>
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<td>St. Dev.</td>
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<td>.4744</td>
<td>.0694</td>
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<tr>
<td>p90-p10</td>
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<tr>
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<td>p50-p10</td>
<td>1.1215</td>
<td>1.9297</td>
<td>1.0799</td>
</tr>
</tbody>
</table>

**Figure:** Distribution of relative prices (top), normalized store sales (center), and annualized sales growth rates (bottom), at the store-product level.

**Source:** IRI Symphony data, 2001-2007.
Regression Results

Definitions: 

\[ \hat{p}_{us,t} \equiv \log P_{us,t} - \frac{1}{N^S_{ut}} \sum_{s=1}^{N^S_{ut}} \log P_{us,t}; \]

\[ \hat{e}_{s,t} \equiv \frac{1}{N^U_{s,t}} \sum_{u=1}^{N^U_{s,t}} \hat{p}_{us,t}; \]

\[
\begin{array}{l}
\Delta \log(Sales_{us}) \\
\log(Sales_s) & -.05674^{**} \\
& (.00075) \\
Age FE & \checkmark \\
Store FE & \checkmark \\
Time FE & \checkmark \\
R^2 & .1371 \\
Obs. (millions) & 43.4 \\
\end{array}
\]

Table: IRI Symphony data (2001-2007). \(s = \text{store}, \ us = \text{UPC within store. All regressions controlled by store-product log-sales. Store age measured as elapsed time since store opening date. SE in parentheses, clustered at the Store \times \text{UPC level. Significance levels: } * = 10\%; ** = 5\%; *** = 1\%.}

(1) Larger stores experience lower per-product growth rates.

(2) Larger stores set higher prices relative to other stores within same product market.

(3) Larger stores are more expensive on average.

(4) Size effects are significant after controlling for store age.
Regression Results

**Definitions:**

\[
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\]

\[
\hat{e}_{s,t} \equiv \frac{1}{N^U_{s,t}} \sum_{u=1}^{N^U_{s,t}} \hat{p}_{us,t};
\]

\[
(1a) \quad \Delta \log(Sales_{us}) \quad (2a) \quad \hat{p}_{us}
\]

<table>
<thead>
<tr>
<th>log(Sales_s)</th>
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<th>(-0.01805^{***})</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(\text{Age FE})</th>
<th>(\checkmark)</th>
<th>(\text{Store FE})</th>
<th>(\checkmark)</th>
<th>(\text{Time FE})</th>
<th>(\checkmark)</th>
</tr>
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</table>

<table>
<thead>
<tr>
<th>(R^2)</th>
<th>(0.1371)</th>
<th>(0.1541)</th>
</tr>
</thead>
</table>

| Obs. (millions) | \(43.4\) | \(59.81\) |

**Table:** IRI Symphony data (2001-2007). \(s = \text{store}\), \(us = \text{UPC within store}\). All regressions controlled by store-product log-sales. Store age measured as elapsed time since store opening date. SE in parentheses, clustered at the Store \(\times\) UPC level. Significance levels: \(* = 10\%\); \(** = 5\%\); \(*** = 1\%\).

1. Larger stores experience lower per-product growth rates.
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3. Larger stores are more expensive on average.
4. Size effects are significant after controlling for store age.
### Regression Results

**Definitions:**
\[
\hat{p}_{us,t} \equiv \log P_{us,t} - \frac{1}{N_{st}} \sum_{s=1}^{N_{st}} \log P_{us,t};
\]
\[
\hat{e}_{s,t} \equiv \frac{1}{N_{s,t}} \sum_{u=1}^{N_{s,t}} \hat{p}_{us,t};
\]

<table>
<thead>
<tr>
<th>( \Delta \log(Sales_{us}) )</th>
<th>( \hat{p}_{us} )</th>
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<tr>
<td>( \log(Sales_{s}) )</td>
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<td>(.01805^{***})</td>
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<td>(.00023)</td>
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<td>(\checkmark)</td>
</tr>
<tr>
<td>Store FE</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>Time FE</td>
<td>(\checkmark)</td>
<td>(\checkmark)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.1371</td>
<td>.1541</td>
</tr>
<tr>
<td>Obs. (millions)</td>
<td>43.4</td>
<td>59.81</td>
</tr>
</tbody>
</table>

**Table:** IRI Symphony data (2001-2007). \( s = \text{store}, \ us = \text{UPC within store. All regressions controlled by store-product log-sales. Store age measured as elapsed time since store opening date. SE in parentheses, clustered at the Store \times UPC level. Significance levels: }^{*}=10\%; \ ^{*{*}}=5\%; \ ^{*{*{*}}}=1\%.**

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Regression Results

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\[ \hat{e}_{s,t} \equiv \frac{1}{N_{s,t}} \sum_{u=1}^{N_{U}} \hat{p}_{us,t}; \]

<table>
<thead>
<tr>
<th></th>
<th>(1a) ( \Delta \log(Sales_{us}) )</th>
<th>(1b) ( \Delta \log(Sales_{us}) )</th>
<th>(2a) ( \hat{p}_{us} )</th>
<th>(2b) ( \hat{p}_{us} )</th>
<th>(3a) ( \hat{e}_{s} )</th>
<th>(3b) ( \hat{e}_{s} )</th>
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<tbody>
<tr>
<td>log( (Sales_{s}) )</td>
<td>-.05674***( (.00075) )</td>
<td>-.05545***( (.00077) )</td>
<td>.01805***( (.00023) )</td>
<td>.01588***( (.00023) )</td>
<td>.01667***( (.00008) )</td>
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<tr>
<td>Age FE</td>
<td>( \times )</td>
<td>( \checkmark )</td>
<td>( \times )</td>
<td>( \checkmark )</td>
<td>( \times )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Store FE</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>Time FE</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
<td>( \checkmark )</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.1371</td>
<td>.1373</td>
<td>.1541</td>
<td>.1550</td>
<td>.8862</td>
<td>.8917</td>
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<tr>
<td>Obs. (millions)</td>
<td>43.4</td>
<td>43.4</td>
<td>59.81</td>
<td>59.81</td>
<td>59.81</td>
<td>59.81</td>
</tr>
</tbody>
</table>

Table: IRI Symphony data (2001-2007). \( s = \) store, \( us = \) UPC within store. All regressions controlled by store-product log-sales. Store age measured as elapsed time since store opening date. SE in parentheses, clustered at the Store \( \times \) UPC level. Significance levels: \( * = 10\%; ** = 5\%; *** = 1\% \).

(1) Larger stores experience lower per-product growth rates.
(2) Larger stores set higher prices relative to other stores within same product market.
(3) Larger stores are more expensive on average.
(4) Size effects are significant after controlling for store age.
**Solution Algorithm**

**Step 1.** Set $k = 0$. Choose guesses $U^{(0)}$ and $\overline{U}^{(0)} \gg U^{(0)}$. Set $U^{B(0)} = \frac{1}{2} \left( U^{(0)} + \overline{U}^{(0)} \right)$.

**Step 2.** For any $k \in \mathbb{N}$ and $n \in \mathcal{N} \equiv \{0, 1, \ldots, \overline{n}\}$, use VFI to find $\{W_n^{(k)} : n \in \mathcal{N}\}$ from:

$$(r + \delta_f)W_n^{(k)} = \pi_n^{(k)} + n\delta_c \left( W_{n-1}^{(k)} - W_n^{(k)} \right) + \max_{x'_{n+1}} \left[ \eta \circ \mu^{-1} \left( \frac{c + rU^{B(k)}}{x'_{n+1} - U^{B(k)}} \right) \left( W_{n+1}^{(k)} - W_n^{(k)} - x'_{n+1} \right) \right]$$

where $\pi_n^{(k)} \equiv n \left( v + (\delta_f + \delta_c)U^{B(k)} \right) - C(n)$. Store the policy functions $\{x'_{n+1}^{(k)} : n \in \mathcal{N}\}$.

**Step 3.** Check free entry condition. Compute:

$$\Delta^{(k)} := \kappa - \eta \circ \mu^{-1} \left( \frac{c + rU^{B(k)}}{x'_1^{(k)} - U^{B(k)}} \right) \left( W_1^{(k)} - x'_1^{(k)} \right)$$

Stop if $\Delta^{(k)} \in [-\varepsilon, \varepsilon]$. Else, bisect $U^{B(k+1)} = \frac{1}{2} \left( U^{(k+1)} + \overline{U}^{(k+1)} \right)$, such that:

1. If $\Delta^{(k)} > \varepsilon$, then $U^{(k+1)} = U^{(k)}$ and $\overline{U}^{(k+1)} = U^{B(k)}$;
2. If $\Delta^{(k)} < -\varepsilon$, then $U^{(k+1)} = U^{B(k)}$ and $\overline{U}^{(k+1)} = \overline{U}^{(k)}$;

and go back to Step 2. with $[k] \leftarrow [k + 1]$.

[See paper for generalization to model with $(z, \varphi)$ shocks]
Computing Stationary Moments

1. Sellers’ entry and exit rates:

\[ \text{EntryRate} = \frac{S_0}{\sum_{n,z} S_n(z)} \sum_{z_0 \in \mathbb{Z}} \pi_z(z_0) \eta(\theta_1(z_0)) ; \quad \text{ExitRate} = \delta_f + \delta_c \frac{\sum_z S_1(z)}{\sum_{n,z} S_n(z)} \]

2. Sales-weighted average markups:

\[ \bar{m} = \sum_{n \in \mathbb{N}} \sum_{z \in \mathbb{Z}} s_n(z) \frac{p_n(z)}{m_{c_n}(z)} ; \quad \text{where } s_n(z) = \frac{np_n(z)}{\sum_{n,z} np_n(z)} \]

3. Average seller size:

\[ \bar{L} = \left( \sum_{n \in \mathbb{N}} \sum_{z \in \mathbb{Z}} \frac{1}{n} L_n(z) \right)^{-1} ; \quad \text{where } L_n(z) := \frac{nS_n(z)}{\sum_{n,z} nS_n(z)} \]

Legend:

1. \( S_0 \) \( \equiv \) Measure of potential entrants.
2. \( S_n(z) \) \( \equiv \) Measure of sellers size \( n \), productivity \( z \).
3. \( m_{c_n}(z) \equiv C(n, z) - C(n - 1, z) \) \( \rightarrow \) Marginal cost of type \( (n, z) \) seller.
Figure: Histograms of targeted moments across simulated economies in the SMM algorithm.

Notes: Dashed line: calibrated value. Solid line: median of the distribution.
# Entry Rate Data

Table: Number of new stores (aged 52 weeks or less) and all existing stores, per year. The entry rate of stores is computed as the ratio of new stores to all stores.


<table>
<thead>
<tr>
<th>Year</th>
<th>New stores</th>
<th>All stores</th>
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<tr>
<td>2001</td>
<td>18</td>
<td>189</td>
<td>9.52%</td>
</tr>
<tr>
<td>2002</td>
<td>11</td>
<td>182</td>
<td>6.04%</td>
</tr>
<tr>
<td>2003</td>
<td>14</td>
<td>172</td>
<td>8.14%</td>
</tr>
<tr>
<td>2004</td>
<td>9</td>
<td>176</td>
<td>5.11%</td>
</tr>
<tr>
<td>2005</td>
<td>13</td>
<td>185</td>
<td>7.03%</td>
</tr>
<tr>
<td>2006</td>
<td>20</td>
<td>187</td>
<td>10.7%</td>
</tr>
<tr>
<td>2007</td>
<td>32</td>
<td>200</td>
<td>16%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>16.71</strong></td>
<td><strong>184.43</strong></td>
<td><strong>8.9%</strong></td>
</tr>
</tbody>
</table>
Micro price statistics, conditional on survival:

1. Instantaneous hazard rate:

\[ h_n(z, \varphi) = \eta(\theta_{n+1}(z, \varphi)) + n\delta_c + \sum_{\tilde{z} \neq z} \lambda_z(\tilde{z}|z) + \sum_{\tilde{\varphi} \neq \varphi} \lambda_\varphi(\tilde{\varphi}|\varphi) \]

2. Frequency of price change:

\[ f_n(z, \varphi) = 1 - \exp\left\{-h_n(z, \varphi)\right\} \]

3. Expected duration of price spells:

\[ d_n(z, \varphi) = \frac{1}{h_n(z, \varphi)} \]

4. Expected size of price changes (letting \( \hat{p} := \ln(p) \)):

\[
\begin{align*}
\hat{p}_n^\Delta(z, \varphi) &= \eta(\theta_{n+1}(z, \varphi)) \left| \hat{p}_{n+1}(z, \varphi) - \hat{p}_n(z, \varphi) \right| + n\delta_c \left| \hat{p}_n(z, \varphi) - \hat{p}_{n-1}(z, \varphi) \right| \\
&\quad + \sum_{\tilde{z} \neq z} \lambda_z(\tilde{z}|z) \left| \hat{p}_n(\tilde{z}, \varphi) - \hat{p}_n(z, \varphi) \right| + \sum_{\tilde{\varphi} \neq \varphi} \lambda_\varphi(\tilde{\varphi}|\varphi) \left| \hat{p}_n(z, \tilde{\varphi}) - \hat{p}_n(z, \varphi) \right|
\end{align*}
\]

Aggregation:

- Use unconditional p.m.f. \( \{g_n(z)\} \).

- For price changes, use renewal p.m.f. (i.e. conditional on price adjustment) \( r_n(z) = \frac{g_n(z)f_n(z)}{\sum_{n,z} g_n(z)f_n(z)} \)
Figure: Hazard rate, frequency, duration, expected price change, and price change dispersion, as a function of size ($n$), in the calibrated economy.
Figure: Responses to aggregate and transitory 1% increase in buyer search cost (c).
Margins of Adjustment (All Shocks)

From $x_n = V^B(n; \{p_n, x_{n-1}, x_{n+1}\})$, price level can be *decomposed* as follows:

$$p_n = \begin{cases} v - rx_n & \text{[0, v]} \quad \text{"Baseline" margin} \\ \delta_f(U^B - x_n) & \leq 0 \quad \text{"Exit" margin} \\ \eta(\theta_{n+1})(x_{n+1} - x_n) & \leq 0 \quad \text{"Growth" margin} \\ n\delta_c \left( \frac{U^B + (n-1)x_{n-1}}{n} - x_n \right) & \leq 0 \quad \text{"Separation" margin} \end{cases}$$

**Figure:** Component-wise response to shocks to marginal costs (left), utility (center), and search costs (right).
Entry Rate when $c \downarrow$

- Entry rate $= \frac{\# \text{ Entrants}}{\# \text{ Incumbents}}$, i.e.

$$\text{Entry Rate} = \eta(\theta_1) \times S_0 \times \left( \sum_{n=1}^{+\infty} S_n \right)^{-1}$$

- When $c \downarrow...$

  1. $\eta(\theta_1) \uparrow \quad \Rightarrow \quad$ Finding 1st customer becomes more likely.

  2. $S_0 / \sum_{n=1}^{+\infty} S_n \downarrow \quad \Rightarrow \quad$ Less customers around to be matched with.